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THE STABILITY ANALYSIS OF NUMERICAL SCHEMES IN COUPLED FLOW AND GEOMECHANICS: PORO-ELASTIC AND PORO-ELASTOPLASTIC BEHAVIOR

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ABSTRACT

We perform a thorough stability analysis of fully coupled and iteratively coupled solution techniques for the numerical solution of coupled flow and geomechanics. We restrict our attention to single-phase flow, but consider elastic as well as elastoplastic material behavior.

We choose pressure and displacement as the primary unknowns, and we discretize the continuum equations in space using a finite volume method (FVM) for the flow problem, and a finite element method (FEM) for the mechanical problem. Implicit backward Euler is used for time discretization. The benefits of the proposed discretization are the following: (1) The choice of primary unknowns is practical and natural in order to bridge between a reservoir simulator and a geomechanics simulator in a sequential fashion. (2) Solution of the flow problem with FVM yields local mass conservation and flux continuity at the element level, a property that does not hold for the commonly-used, nodal-based FEM discretizations. (3) The FVM can be seen as a mixed finite element method (MFEM) with inexact quadrature; since the pressure approximation is piecewise constant (one order lower than the displacement), this scheme enjoys enhanced spatial stability properties during fast transients and for quasi-incompressible systems. (4) An implicit time discretization is mandatory for the solution of the pressure equation with a reasonable time step size if the fluid/rock system is only slightly compressible.

Two iteratively coupled strategies are considered: the drained split and the undrained split. We use the Von Neumann method to obtain stability estimates for the discretization described above, and the three coupling methods (drained, undrained, and fully coupled). The main results of the stability analysis are the following: (1) The drained split, which is typically used in the oil industry, is conditionally stable. Its stability condition is *independent of time step size*, and only dependent on the strength of the coupling between flow and mechanics. This implies that problems with strong coupling cannot be solved by the drained method, in contrast with the widespread belief that such problems can be handled by reducing the time step size. (2) Oscillations in the solution by the drained method can occur even in its range of stability. Such oscillations are associated with negative amplification factors in the Von Neumann stability analysis. (3) The fully coupled and undrained methods are unconditionally stable regardless

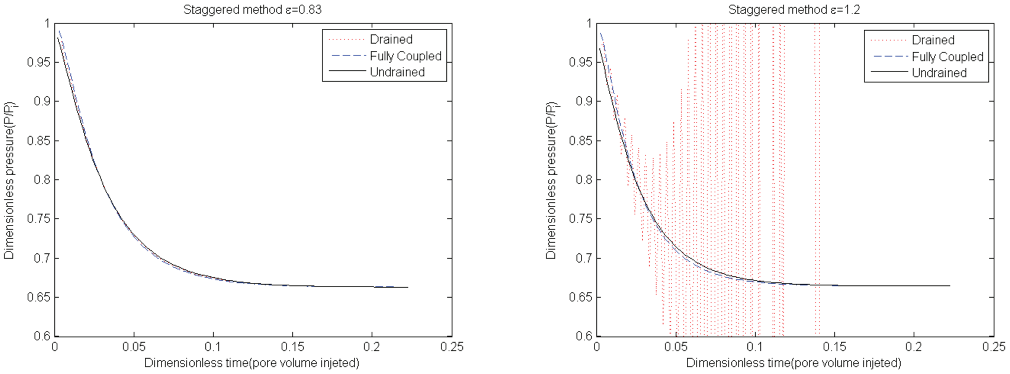

Figure 1: Simulation results for a poroelastic problem with the fully-coupled, drained, and undrained methods. Left: coupling strength $\epsilon = 0.83$. Right: coupling strength $\epsilon = 1.2$. The drained method is unstable if $\epsilon > 1$.

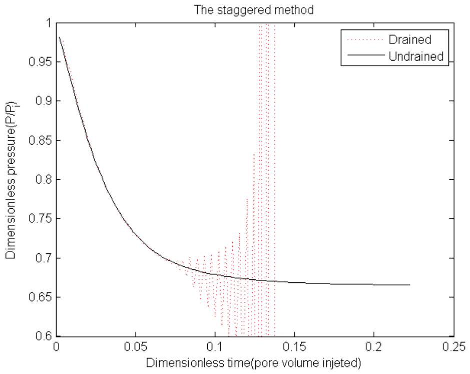

Figure 2: Simulation results for a poroelastoplastic problem with the drained, and undrained methods. The drained method is stable initially, but loses stability as the material enters the inelastic regime—independently of time step size.

of the coupling strength, and they do not suffer from oscillations in time. (4) Plasticity increases the coupling strength between flow and mechanics. This is an important result: if one uses the drained split, stability may be lost during the simulation (as part of the system enters the inelastic regime) *regardless of time step size*.

We confirm the results of the Von Neumann stability analysis by numerical simulation. We present numerical examples in 1D and 2D, both in the linear elastic regime and the nonlinear inelastic regime. As an illustration, in Figure 1 we show the evolution of pressure for a representative simulation of linear poroelasticity. The drained method is unstable if the coupling strength exceeds one (see caption). In Figure 2 we show results for a nonlinear elastoplastic problem. The drained method is stable initially, but loses stability as the material enters the inelastic regime—independently of time step size.

In conclusion, the undrained method is strongly recommended in reservoir simulation coupled with geomechanics, since it is unconditionally stable, and the computational costs of the drained method and the undrained method are the same. We strongly discourage the commonly-used drained split, because stability depends on coupling strength only, and cannot be recovered by decreasing the time step size.

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