

# AN UNCONDITIONALLY STABLE SPACE–TIME DISCONTINUOUS GALERKIN METHOD FOR LINEAR THERMO-ELASTO-DYNAMICS WITH PROPAGATING WEAK DISCONTINUITIES

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## ABSTRACT

The study of deformations in dynamically and thermally loaded elastic solids has important applications in engineering. A wide literature exists on the numerical solution of dynamic thermo-elasticity problems via finite element methods (FEM). These methods include Laplace-transform FEMs and semi-discrete methods, whereby finite elements are used in space and finite differences are used in time. The main drawback of Laplace-transform FEMs is the necessity for numerical Laplace inversion while for semi-discrete methods numerical stability is highly dependent on the time integration scheme used. Motivated by the excellent stability properties of discontinuous Galerkin (DG) methods (see [1] for a comprehensive analysis of the stabilization mechanisms in DG methods), in this talk we focus on the formulation of an unconditionally stable space–time DG finite element method (FEM) for thermo-elasto-dynamics. This FEM method is based on the extension of an existing method for linear elasto-dynamic problems.

DG methods have received considerable attention in many application areas, such as computational fluid dynamics, chemical transport problems, and other problems with convection [2]. Within DG methods, one also finds space–time DG methods for almost every problem with a time-evolving physical phenomenon, including elasto-dynamics. Some of the early work in this field is that in [3] and [4] in which numerical stability was achieved by the use of least square terms. More recently, stability in the context of space-time DG methods was discussed in [5], in which a conditionally stable method is proposed but written as a symmetric system, and in [6] who have proposed a formulation based on that in [4] but revised to achieve unconditional stability without the use of least square terms and applicable to unstructured grids. A very recent and interesting space-time DG formulation has been proposed in [7] and framed in the context of general manifold theory. This formulation allows for displacement discontinuities across every interelement boundary, is applicable with completely unstructured grids, and has good shock capturing capabilities.

DG *space-time* FEMs have been developed for static (i.e., neglecting inertia) and dynamic coupled thermoelasticity problems in [8] and [9], respectively. In [8], an adaptive strategy based on a dual problem

for a static linear thermoelastic problem is presented for a space-time finite element method, which is based on a standard Galerkin method in space with piecewise linear displacement and temperature interpolations and a DG method in time with piecewise constant approximations. In [9] one finds a different adaptive space-time FEM, based on the residual approach and the adjoint state of the linear fully-coupled thermo-elasto-dynamic problem.

In this talk a DG space-time FEM is presented that can correctly represent moving and stationary singular surfaces in boundary value problems of fully coupled thermo-elasto-dynamics. For isothermal elasto-dynamics, the numerical solution for propagating phase boundaries in solids using a space-time DG FEM has been presented in [6]. The formulation in [6] has been recently extended in [10] to dynamic fully coupled thermo-elasto-dynamic problems with propagating phase transition interfaces. The proposed formulation is shown to be consistent and unconditionally numerically stable. The key aspect of the paper is that numerical stability is achieved by a physically-based approach consisting in enforcing the jump conditions of the balance of linear momentum and the balance of energy. The method's convergence rates are demonstrated using some exact solutions of two- and three-dimensional space-time boundary value problems. Solutions to boundary value problems with moving discontinuities are presented in which the degree of thermo-mechanical coupling is varied to show how the system's response in terms of stress, temperature, and energy release rate is affected.

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