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EQUATIONS FOR THE RESPONSE PROBABILITY DENSITY FOR A DYNAMIC SYSTEM UNDER N-COMPONENT NON-POISSON IMPULSE PROCESS EXCITATION

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ABSTRACT

State vector of a dynamic system under a Poisson train of impulses is a non-diffusive Markov process and its joint probability density function satisfies an integro-differential generalized Fokker-Planck-Kolmogorov equation which is also called Kolmogorov-Feller equation. If the train of impulses is driven by non-Poisson, for example renewal, counting processes, the state vector is not a Markov process. Non-Markov pulse problems can be converted into Markov ones by augmenting the state vector of the dynamic system by auxiliary variables driven by Poisson processes. Exact techniques of this kind have been developed for trains of impulses driven by Erlang renewal processes (Iwankiewicz and Nielsen, 1999), by a generalized Erlang renewal process (Iwankiewicz, 2002) and by some classes of non-Erlang renewal processes (Tellier and Iwankiewicz, 2005) . For these classes of random impulse processes the techniques of equations for response moments have been developed. Recently the explicit equations governing the response probability density have been derived for oscillators under random trains of impulses driven by single non-Poisson processes (Iwankiewicz, 2005, 2008).

The excitation considered in the present paper is a number of $\mathbf{n}$ random trains of impulses, each of whom is driven by a non-Poisson, renewal process. Each of these processes is a renewal process with inter-arrival times being the sum of two independent negative-exponential distributed random variables. All component impulse processes are assumed to be statistically independent. Each of the original impulse processes is recast into a Poisson driven impulse process, with the aid of an auxiliary, purely jump zero-one valued stochastic variable. Hence there are $\mathbf{n}$ additional state variables. Each auxiliary variable is governed by the stochastic differential equation driven by two independent Poisson processes, with different parameters ν, μ. It is a crucial fact that each auxiliary variable is tantamount to two Markov states. The Markov chain for the whole problem is constructed by considering the coincidences of the states of the individual jump processes. Thus the total number of Markov states equals $\mathbf{2^n}$. The jump probability intensity functions are determined for all state variables and for all possible jumps. An arbitrary, non-linear, single-degree-of-freedom dynamic system under purely external excitation is

considered. The equations governing the joint probability density-distribution function of the response and of the Markov states of the auxiliary variables are derived from the general integro-differential forward Chapman-Kolmogorov equation (Gardiner 1985) with the aid of those jump probability intensity functions. The resulting equations form a set of integro-partial differential equations.

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