

# PARALLEL IMPLEMENTATION OF A PREDICTOR-CORRECTOR SCHEME FOR THE SOLUTION OF THE NAVIER-STOKES EQUATIONS

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## ABSTRACT

This paper presents a parallel implementation of a predictor-corrector scheme to solve the Navier-Stokes equations. This scheme consists in solving the momentum and continuity equations consecutively in order to converge to the monolithic solution. That is no fractional error is introduced. In addition, this scheme is very well suited for massively parallel implementation. In fact, one can use simple preconditioners to solve the non-symmetric momentum equations (GMRES) and to solve the symmetric continuity equation (CG). This gives good scalability properties of the algorithm. The implementation of the mesh partitioning technique is presented, as well as the parallel performance and scalability for thousands of processors.

A straightforward way to solve the Navier-Stokes equations is to consider the monolithic scheme, that is to solve the momentum and continuity equations at the same time. However, it is well known that iterative algebraic solvers (GMRES) converge poorly for these coupled equations, unless a good preconditioner is used (like ILU). However, these preconditioners have bad scalability properties and therefore preclude their use on large scale computers (thousands of CPUs). On the one hand, traditional fractional steps techniques split the Navier-Stokes equations and solve consecutively for the momentum and continuity equations. However, they introduce errors due the the splitting. On the other hand, predictor-corrector methods converge to the monolithic solution, while inheriting the good properties of the splitting of the momentum and continuity equations.

The objective of the work is to study mesh partitioning technique for the solution of a predictor-corrector scheme to solve the Navier-Stokes equations. The numerical scheme consists of a finite element method using the variational subgrid scale concept for subgrid scale modeling and stabilization. Next, we present the mesh partitioning technique implemented to solve the algorithm in parallel. In particular, we will point out the important aspects to take into account in order to obtain good parallel behavior on parallel supercomputers. This includes: automatic mesh partitioning for hybrid meshes; communication scheduling; data organization. Also, the parallel performance of the code will be illustrated through its analysis via a performance visualization tool. These kind of tools enables one to graphically identify communication patterns and possible lack of load balance. See Figure 1. In addition, results of an

automatic performance analysis code will be given to catch the global characteristics of the code: L2 cache misses, and instructions per cycle indicator (IPC). Finally, some scalability results obtained on Marenostrum supercomputer using up to 5000 CPU's are given, as shown in Figure 2.

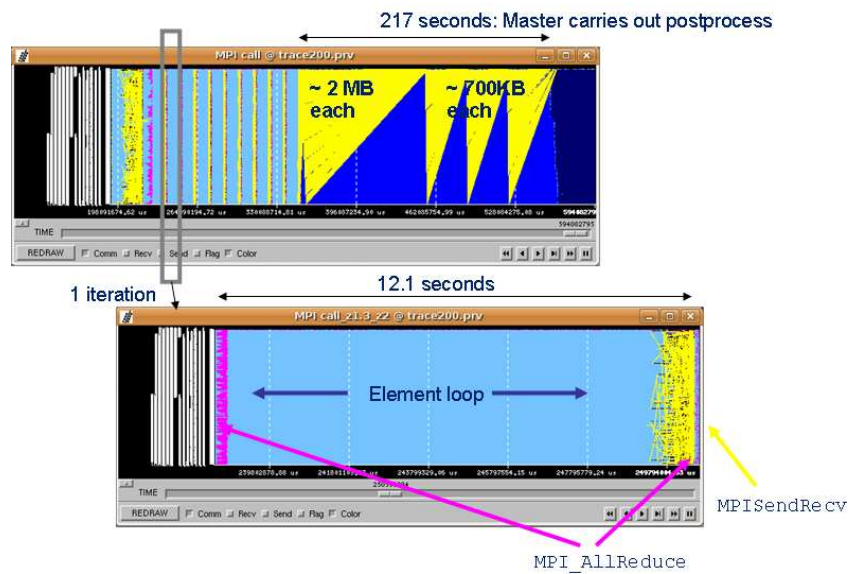


Figure 1: Performance analysis with graphical tools.

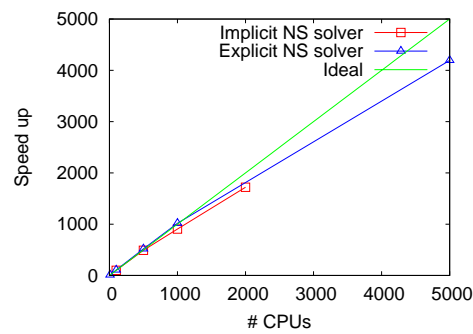


Figure 2: Scalability results.

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