8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

CHATTERING: A NOVEL ROUTE TO CHAOS IN CAM-FOLLOWER IMPACTING SYSTEMS

*** R. Alzate[1], M. di Bernardo[2] and P. T. Piiroinen[3]**

[1] University of Naples - FEDERICO II, Italy
Department of Systems and Computer Science
DIS building, via Claudio 21, 80125, Naples (IT)
r.alzate@unina.it; http://wpage.unina.it/r.alzate

[2] University of Naples - FEDERICO II, Italy / University of Bristol - U.K.
Department of Systems and Computer Science
DIS building, via Claudio 21, 80125, Naples (IT)
mario.dibernardo@unina.it; http://www.sincro.unina.it

[3] National University of Ireland - Galway
Department of Mathematical Physics
Room C304, Áras de Brún, Galway (IRL)
petri.piiroinen@nuigalway.ie

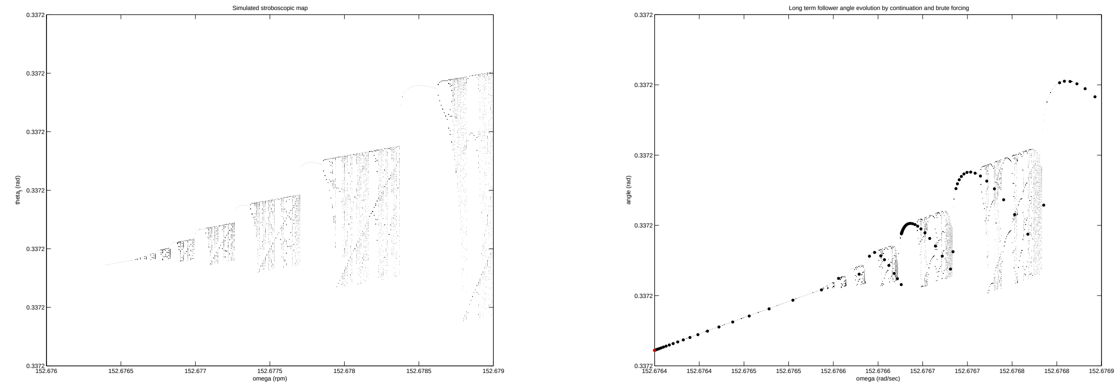
Key Words: *Bifurcations, Cam-follower Systems, Chattering, Grazing, Impact Oscillators, Nonlinear Dynamics, Numerical Continuation, Local Analysis, Piecewise-smooth, Simulation.*

ABSTRACT

The analysis of the nonlinear dynamics associated with simple models of forced harmonic oscillators has been shown to be a powerful tool for the explanation of complex behaviour and bifurcation phenomena in piecewise-smooth dynamical systems (PWS). In particular, the parameter dependence of the dynamics exhibited by single degree of freedom (SDOF) impact oscillators has been studied in detail [1][2][3], unveiling interesting transitions to chaos and related phenomena as grazing [4] and chattering [1]. Grazing (or zero velocity impact) has proven to be the fundamental mechanism to explain. for example, routes to chaos characterized by a sequence of period-adding windows. Indeed such transitions are well explained in terms of the so-called *grazing bifurcation*, a discontinuity induced event by which appropriate theoretical tools have been developed as reported in [4][5]. In addition, a closely related phenomenon depending on sticking annihilation, is predicted to occur when (robust) complete-chattering orbits suddenly jump into chaotic regimes under slight parameter variations [1].

The main aim of this paper is to present, a combination of numerical and analytical tools to characterize the nonsmooth transition to chaos observed in a SDOF model of cam-follower impacting system [8], with particular attention to the complex scenario observed at the interruption of complete chattering sequences.

An accurate numerical simulator has been adapted for multi-impacting (complete chattering) trajectories of a hybrid piecewise-smooth model with a Newton's coefficient of restitution rule [6][7]; analogously, brute-force Montecarlo simulations have been performed to uncover the main bifurcation zones detected when varying the cam rotational-speed ω [see Figure 1.(a)]. Also, numerical continuation routines have been implemented for tracing smooth and non-smooth solution branches [see Figure 1.(b)]. The numerical and analytical local analysis close to the chattering interruption point yields the derivation of a low-dimensional piecewise-smooth map. Further analysis allows the construction of the corresponding discontinuity map (DM) and Poincaré section characterizing the observed dynamical scenario. Hence, the analytical explanation is presented of a novel route to chaos in cam-follower impacting systems due to the transition from complete to incomplete chattering.



(a) (b)

Figure 1: Numerical analysis of bifurcation behaviour for the multi-impacting orbit: (a) stroboscopic map showing a chain of interrupted period-doubling windows, (b) zoomed region showing superposition of solution branch predicted by numerical continuation (black dots).

REFERENCES

[1] C. Budd and F. Dux. “Chattering and related behaviour in impact oscillators”. *Philosophical transactions: physical sciences and engineering*, Vol. **347**, N^o 1683, 365–389, 1994.

[2] C. Budd, F. Dux and A. Cliffe. “The effect of frequency and clearance variations on single-degree-of-freedom impact oscillators”. *Journal of sound and vibration*, Vol. **184**, N^o 3, 475–502, 1995.

[3] A. Nordmark, and R. Kisitu. “On chattering bifurcations in 1 DOF impact oscillator models”. *Royal Institute of Technology, Sweden*, Preprint. 2003.

[4] A. B. Nordmark. “Non-periodic motion caused by grazing incidence in impact oscillators”. *Journal of Sound and Vibration*, Vol. **2**, 279–297, 1991.

[5] M. di Bernardo, C. J. Budd, A. R. Champneys and P. Kowalczyk. *Bifurcation and Chaos in Piecewise Smooth Dynamical Systems – Theory and Applications*, Springer–Verlag, U.K. to be published.

[6] P. T. Piiroinen, and Y. A. Kuznetsov. “An event driven method to simulate Filippov systems with accurate computing of sliding motions”. *ACM Transactions on Mathematical Software*, Vol. **34**, N^o 3, to appear.

[7] A. Nordmark, and P. T. Piiroinen. “Simulation and stability analysis of impact systems with complete chattering”. *In progress*.

[8] R. Alzate, M. di Bernardo, U. Montanaro, and S. Santini. “Experimental and numerical verification of bifurcations and chaos in cam-follower impacting systems”. *Nonlinear Dynamics - Springer. The Netherlands*, Vol. **50**, N^o 3, 409–429, 2007.