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Third-order residual-based scheme for computing inviscid and viscous flows on unstructured grids

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ABSTRACT

A so-called residual-based (RB) scheme has been previously developed [1] for computing compressible flows governed by the Euler or Navier-Stokes equations on structured grids; second, third and higher-order versions of the scheme were successfully applied to inviscid/viscous steady/unsteady flows, with good shock capturing properties for transonic or supersonic flows. The present contribution describes the extension of a third-order RB scheme to general unstructured grids using a finite volume (FV) framework.
The FV discretization of the Euler system $w_t + \nabla \cdot F^E(w) = 0$ is expressed as :

$$\frac{\Delta w_i^n}{\Delta t_i^n} + \frac{1}{|\Omega_i|} \sum_{k \in \mathcal{I}(\Omega_i)} \sum_g \omega_g \left(\mathcal{H}_{i,k}^E\right)_g^n |\Gamma_{i,k}| = 0, \tag{1}$$

with n the time step counter, $\Delta w^n = w^{n+1} - w^n$, w_i the state at the center of the control cell Ω_i, defined by a set $\mathcal{I}(\Omega_i)$ of faces $\Gamma_{i,k}$. The numerical flux $(\mathcal{H}_{i,k}^E)_g$ approximates the normal convective flux at Gauss-point g on the face $\Gamma_{i,k}$, with an associated quadrature weight ω_g. The numerical flux defining the RB scheme takes the following form:

$$(\mathcal{H}_{i,k}^E)_g = \left(\mathcal{H}_{i,k}^c\right)_g - (d_{i,k})_g = \mathcal{H}^c((w_{i,k}^L)_g, (w_{i,k}^R)_g; \underline{n}_{i,k}) - d_{i,k} \tag{2}$$

where $\left(\mathcal{H}_{i,k}^c\right)_g$ is a non-dissipative approximation of the physical normal flux vector with $\mathcal{H}^E$ a simply centered formula and $(w_{i,k}^L)_g$, $(w_{i,k}^R)_g$ the reconstructed states on the left and right sides of the interface $\Gamma_{i,k}$ computed at the Gauss-point g on that face, with the unit normal vector $\underline{n}_{i,k}$ pointing from cell i to the neighboring cell $o(i,k)$ sharing $\Gamma_{i,k}$. For a 3rd-order formulation, states $w_{i,k}^{L/R}$ are computed at each Gauss-point with a quadratic reconstruction taken from [2] :

$$\begin{aligned}(w_{i,k}^{L/R})_g = w_{i/o(i,k)} \quad & +((1-\sigma_{i/o(i,k)})\, \phi_{i/o(i,k)} + \sigma_{i/o(i,k)})\, (\mathbf{r}_g - \mathbf{r}_{i/o(i,k)})^T \cdot \nabla w_{i/o(i,k)} \\ & +\tfrac{1}{2}\sigma_{i/o(i,k)}\, (\mathbf{r}_g - \mathbf{r}_{i/o(i,k)})^T \cdot \mathbf{H}_{i/o(i,k)} \cdot (\mathbf{r}_g - \mathbf{r}_{i/o(i,k)})\end{aligned} \tag{3}$$

with $\mathbf{r}_g$, $\mathbf{r}_{i/o(i,k)}$ the respective positions of the of the Gauss-point g and the left or right cell centroid C_i, $C_{o(i,k)}$, $\nabla w_{i/o(i,k)}$ and $\mathbf{H}_{i/o(i,k)}$ respectively a second-order estimate of the

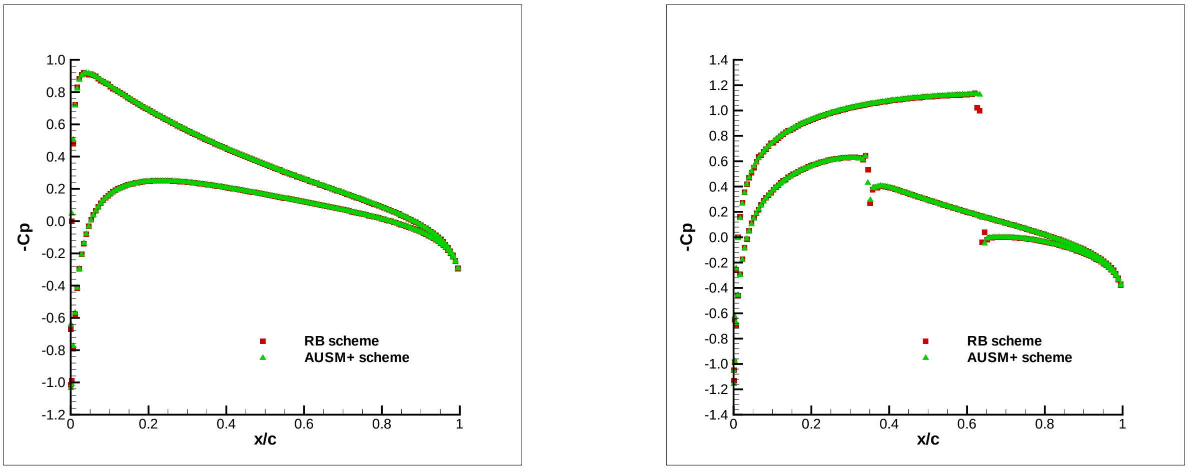
cell-center gradient and a first-order estimate of the Hessian of the solution at the left or right cell centroid, both obtained using a least-square formula on an extended support. The Venkatakrishnan's limiter ϕ is completed with a binary detector σ such that $\sigma = 1$ (quadratic reconstruction) if no high gradient is detected while $\sigma = 0$ (limited linear reconstruction) in high gradient region. The key ingredient of the scheme is the RB dissipation $(d_{i,k})_g$ computed at any Gauss-point g of face $\Gamma_{i,k}$ as :

$$(d_{i,k})_g = d_{i,k} = \frac{1}{2}\, ||\overrightarrow{C_i C_{o(i,k)}}||\, \Phi_{i,k}\, \mathcal{R}_{i,k} \tag{4}$$

where $\Phi_{i,k}$ is a matrix coefficient of order $O(1)$ ensuring the scheme's dissipation [1] and $\mathcal{R}_{i,k}$ is a discrete form of the residual computed on a shifted cell $\Omega_{i,k}$ enclosing face $\Gamma_{i,k}$:

$$R_{i,k} = \frac{1}{|\Omega_{i,k}|} \int_{\Omega_{i,k}} \nabla \cdot F^E(w)\, d\Omega. \tag{5}$$

This third-order RB dissipation adapts itself to the problem considered : when solving the Navier-Stokes equations $w_t + \nabla \cdot (F^E - F^V) = 0$, the flux F^E in (5) is replaced with the total flux $F^E - F^V$. It is computed from cell-centered states and second-order estimates of node values. Detailed comparisons, in terms of both accuracy and efficiency, between the RB scheme and conventional upwind schemes (Roe, AUSM+, HLLC ...) relying on quadratic reconstruction will be provided at the Conference for 2D and 3D external aerodynamic problems. The preliminary results presented below illustrate the ability of the third-order RB scheme to yield accurate solutions of 2D inviscid flows.



(a) subsonic case($M_\infty = 0.5$, $\alpha = 2$).

(b) transonic case($M_\infty = 0.8$, $\alpha = 1.25$).

Figure 1: Inviscid flow over a NACA0012 airfoil. Pressure coefficient distributions computed using the third-order RB scheme and third-order AUSM+ scheme. (a) lift coefficient for the subsonic case : $C_L^{AUSM+} = 0.282$, $C_L^{RB} = 0.280$. (b) lift and drag coefficients for the transonic case : $C_L^{AUSM+} = 0.345$, $C_L^{RB} = 0.341$; $C_D^{AUSM+} = 0.0223$, $C_D^{RB} = 0.0221$.

References

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