

Generic implementation of Galerkin methods including partition of unity enrichment

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ABSTRACT

The finite element formulation is derived from the more general Galerkin numerical method. Assuming the solution space to take the form of Lagrange polynomials allows various simplifications and optimisation in the code, while retaining a high level of generality [1]. The emergence of XFEMs has however demonstrated that for specialised applications, such as the simulation of strong and weak discontinuities or singularities, enriched Galerkin methods can be better suited than the classical FEM approach [4-7].

However, the implementation of new shape functions and integration schemes as well as mesh-geometry interaction procedures [1-3] represents a significant investment in time, especially if no exact integration scheme exists, or elements are high-order and multidimensional. The implementation of a finite element library, using object-oriented design helps separate functionally orthogonal modules. We have implemented a virtual machine which allows the decoupling between geometrical and numerical aspects, as well as perform numerical integration and derivation on arbitrary functions. When exact integration scheme cannot be implemented, the integrated Delaunay tessellator can be used to automatically generate adapted Gauß points [1]. This architecture allows the easy implementation of novel enrichment schemes, as well as general Galerkin methods.

As examples, we solve a mesh made from struts with harmonic shape functions and a vibrating cracked membrane using Bessel functions, and partition of unity enrichment.

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