

Variational Multiscale Methods in Heat Transfer and Fluid Mechanics[†]

David K. Gartling

Engineering Sciences Center
Sandia National Laboratories
Albuquerque, New Mexico USA
dkgartl@sandia.gov

Key Words: *Multiscale Methods, Finite Elements, Heat Transfer, Fluid Mechanics.*

ABSTRACT

Variational Multiscale Methods (VMS), as introduced by Hughes, *et al.* [1,2] have been employed and demonstrated for a variety of applications in computational mechanics. The basic idea of VMS is to decompose the macroscopic mechanics problem into a series of problems each of which represents an appropriate length and time scale for the problem of interest. When cast in a computational framework using a weighted residual approach, some of the problem scales will be resolvable on the computational grid and some will need to be solved and/or modeled as subgrid quantities. It is the use and specification of the subgrid quantities that determine how the VMS method is viewed and categorized. To date, three main variants of the method have been developed. For some applications, the subgrid quantities provide a method for stabilization of the basic numerical method through residual redistribution [1]. The implementation of physical models, such as turbulence, is appropriately and conveniently accomplished through a VMS approach [3]. Finally, the method can be utilized as a paradigm for coupling various types of methods, such as the joining of material regions with continuum and non-continuum behavior [4].

In the present work the VMS methodology is applied in three different applications with the subgrid scales being used primarily to either incorporate physical models or provide a coupling mechanism. The acoustically filtered form of the Navier-Stokes equations is formulated utilizing a finite element, VMS approach to allow the implementation of an LES turbulence model. The interest for this computational capability is in the simulation of non-Boussinesq flows. A second application of VMS is in the development of a stochastic model for heat conduction in a reactive material. In this case, the subgrid scale is used to represent the randomized, heterogeneous material and its chemically reactive response to thermal perturbations. The third application of VMS provides a method for coupling a continuum, compressible flow code with a non-continuum, particle-based method. The application of interest involves the expansion of a plasma in a complex geometry.

REFERENCES

- [1] T.J.R.Hughes. “Multiscale phenomena: Green’s functions, the Dirichlet-to-Neumann formulation, subgrid scale models, bubbles and the origin of stabilized methods”. *Comp. Meth. Appl. Mech. Eng.*, Vol. **127**, 387–401, 1995.
- [2] T.J.R.Hughes, G. R. Feijóo, L. Mazzei and J-B. Quincy. “The variational multiscale method - A paradigm for computational mechanics”. *Comp. Meth. Appl. Mech. Eng.*, Vol. **166**, 3–24, 1998.
- [3] T.J.R.Hughes, L. Mazzei and K.E. Jansen. “Large eddy simulation and the variational multiscale method”. *Comput. Visual. Sci.*, Vol. **3**, 47–59, 2000.
- [4] G. Wagner and W.K. Liu. “Coupling of atomistic and continuum simulations using a bridging scale decomposition”. *J. Comput. Phys.*, Vol. **190**, 249–274, 2003.

† Sandia National Laboratories is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the U. S. Department of Energy’s National Nuclear Security Administration under contract DE-AC04-94AL85000