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A 2D FE APPROACH FOR NONLOCAL ELASTIC PROBLEMS

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ABSTRACT

The key idea of nonlocal approaches to elastic mechanical problems is to use a continuum formulation endowed with information regarding the material behaviour at microstructural level. To this aim, an internal length material scale (say ℓ_0), driving the modelling of the diffusion processes involving neighbouring points linked together by long range forces, is introduced (see e.g. [1]).

The present study refers to integral type nonlocal elasticity as the one envisaged by Eringen and co-workers [2]. It refers to linear homogeneous isotropic continua and it is characterized by a stress-strain relation of convolutive-type containing an *attenuation function*, $A(\boldsymbol{x},\boldsymbol{x}')$, aimed at capturing the diffusion process of the nonlocality effects. In particular, the assumed constitutive equation is:

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \xi_1 \mathbf{D} : \boldsymbol{\varepsilon}(\boldsymbol{x}) + \xi_2 \int_V A(\boldsymbol{x},\boldsymbol{x}')\mathbf{D} : \boldsymbol{\varepsilon}(\boldsymbol{x}')\mathrm{d}V', \quad \forall \boldsymbol{x} \in V, \quad (\xi_1 + \xi_2 = 1)$$

where: ξ_1 and ξ_2 denote the volume fractions of material complying with local and nonlocal elasticity, respectively; $\mathbf{D}$ is the tensor of classical isotropic elasticity.

In this paper a numerical technique, first proposed in [3], as nonlocal finite element method (NL-FEM), is implemented to solve 2D nonlocal elastic problems. It is worth noting that the NL-FEM leads to a solving equation system formally equal to that pertaining to the standard FEM, but with the relevant global (nonlocal) stiffness matrix reflecting all the nonlocality features of the problem. The solving linear equations system can in fact be given in the shape:

$$\hat{\mathbf{K}}\mathbf{U} = \mathbf{F} \quad \text{with} \quad \hat{\mathbf{K}} = \xi_1 \sum_{n=1}^{N_e} \mathbf{C}_n^{\mathrm{T}} \mathbf{k}_n^{loc} \mathbf{C}_n + \xi_2 \sum_{n=1}^{N_e} \sum_{m=1}^{N_e} \mathbf{C}_n^{\mathrm{T}} \mathbf{k}_{nm}^{nonloc} \mathbf{C}_m \,,$$

where: $\mathbf{C}_n$ and $\mathbf{C}_m$ denote the connettivity matrices; $\mathbf{k}_n^{loc}$ and $\mathbf{k}_{nm}^{nonloc}$ are the element local and nonlocal stiffness matrices, respectively. Each finite element (FE), besides the standard element stiffness matrix $\mathbf{k}_n^{loc}$, is endowed with a direct- or self-stiffness matrix, $\mathbf{k}_{nn}^{nonloc}$, plus a set of indirect- or cross-stiffness matrices, $\mathbf{k}_{nm}^{nonloc}$, strictly related

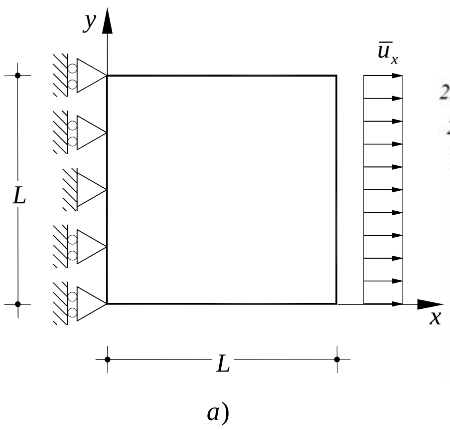
to the mesh geometry, and containing information coming from the other FEs.

The cross-stiffness matrix is in fact given by:

$$\mathbf{k}_{nm}^{nonloc} = \int_{V_n}\int_{V_m} A(\boldsymbol{x},\boldsymbol{x}')\mathbf{B}_n^{\mathrm{T}}(\boldsymbol{x}):\mathbf{D}:\mathbf{B}_m(\boldsymbol{x}')\mathrm{d}V'\mathrm{d}V$$

and, in practice, it vanishes for FEs too far from each other with respect to an influence distance, i.e. the maximum distance beyond which the nonlocality effects are almost negligible ($A(\boldsymbol{x},\boldsymbol{x}')\cong 0$). Thus, the global stiffness matrix, which is symmetric and positive semi-definite, turns out to be banded but with a bandwidth in general larger than in the standard FEM.

The NL-FEM has been applied to analyze a nonlocal elastic square plate as the one shown in Figure 1a. The plate is subjected to uniform prescribed displacements $\bar{u}_x = 0.01$ cm while $E = 2.1\times 10^6$ daN/cm^2; $L = 5$ cm; and the thickness $t = 0.5$ cm. Numerical analyses have been performed assuming a bi-exponential attenuation function. The plate has been discretized using a uniform mesh of 30×30 eight-nodes isoparametric elements, carrying on the numerical integration by means of Gauss quadrature with 9 points per element. Figure 1b displays a 3D plot of the strain distribution $\varepsilon_x(x,y)$ obtained for $\ell_0 = 0.1$ cm and $\xi_2 = 0.5$. As expected, the uniform local elastic solution is recovered in the core domain, while an increasing trend of the strains is detected close to the edges.



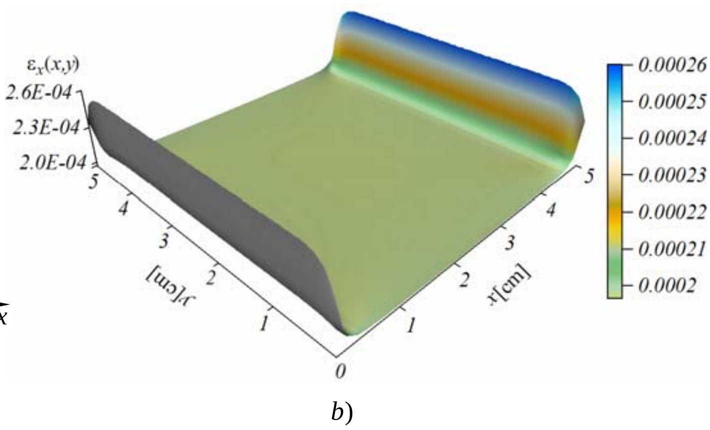


Figure 1: a) Plate subjected to uniform prescribed displacements; b) Strain distribution in the *x* direction.

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