

## Stability and Sensitivity of Shell-Like Structures Considering Imperfections and Contact

\* K. Schweizerhof, E. Ewert

Institut für Mechanik  
Universität Karlsruhe

Universität Karlsruhe  
76131 Karlsruhe, Germany  
ks@ifm.uni-karlsruhe.de

**Key Words:** *structural stability, sensitivity, shell buckling.*

### ABSTRACT

In standard stability investigations of structures applying the finite element method usually bifurcation and snap-through loads – so-called *singular* or *stability points* – are detected, see e.g. [4],[6],[7]. It is well-known that for complex structures like cylindrical shells these loads depend strongly on the geometrical and other imperfections, see [8],[9],[10]. It has been shown in [1] that the singular points and the corresponding modes vary significantly for small deviations of the approximated geometry using different ansatz-functions. Furthermore investigations in [1] have shown that the converging behavior for imperfect structures is non-monotone. Particular limitations exist for the stability analysis of shell structures which involve contact between parts or with surrounding objects. In the end it could be followed that stability points and the corresponding loading determined numerically by static stability analysis are often of limited use for design purposes.

In contrast to the loading obtained for singular points the so-called post-buckling loads, which are the stable equilibrium states in the post-buckling region, are rather independent of geometrical imperfections and of approximation order, see [1]. Therefore it is advantageous to use post-buckling loads for design purposes instead of loads obtained for singular points. Since the applicability of static analyses in the computation of post-buckling paths like proposed e.g. in [5] is rather limited, it is favorable to model the complete loading and deformation behavior by a time dependent process, see [3], [2], [11], [12]. The major advantage of a purely transient analysis is the complete simulation of the buckling process as it happens in reality. This is possible with moderate numerical effort, since the matrices used in the solution are usually better conditioned compared to pure static analysis. In addition, this allows to take the changing boundary conditions as found in contact situations properly into account.

For practical design purposes not only the equilibrium state itself is significant but also the "robustness" of such states against finite perturbations in contrast to infinitesimal perturbations. In the case of systems with many equilibrium states at a defined load level finite perturbations can transfer the mechanical system from a stable equilibrium state to another equilibrium state or in some cases even to

an unbounded motion initiating buckling. Then a sensitivity measure can be defined as the reciprocal value of the minimum perturbation energy, necessary for this transfer  $S = 1/W_{per,min}$ , see [2].

In the present paper stability and sensitivity studies are performed for simple stability problems (beams) and finally for rather general shell structures involving geometrical imperfections and in particular contact.

## REFERENCES

- [1] E. Ewert, K. Schweizerhof. "Numerical Aspects in the Computation of Singular Points /Modes for Cylindrical Shells." *Procs. ANASS-Workshop*, Sept. 26-28, 2007, Zagreb.
- [2] E. Ewert, K. Schweizerhof, P. Vielsack. "Measures to judge the sensitivity of thin-walled shells concerning stability under different loading conditions." *Special Issue of Comp. Mech.*, Vol. **37** (6), 507–522, 2006.
- [3] E. Riks, C.C. Rankin, F.A. Brogan. "On the solution of mode jumping phenomena in thin-walled shell structures." *Comp. Meth. Appl. Mech. Eng.*, Vol. **136**, 59–92, 1996.
- [4] P. Wriggers, W. Wagner, C. Miehe. "A quadratically convergent procedure for the calculation of stability points in finite element analysis." *Comp. Meth. Appl. Mech. Eng.*, Vol. **70**, 329–347, 1988.
- [5] W. Wagner, P. Wriggers. "A simple method for the calculation of postcritical branches." *Eng. Comp.*, Vol. **5**, 103–109, 1988.
- [6] H.A. Mang, Ch. Schranz, P. Mackenzie-Helnwein. "Conversion from imperfection-sensitive into imperfection-insensitive elastic structures I: Theory." *Comp. Meth. Appl. Mech. Eng.*, Vol. **195**: **13-16**, 1422–1457, 2006.
- [7] H.A. Mang, Ch. Schranz, P. Mackenzie-Helnwein. "Conversion from imperfection-sensitive into imperfection-insensitive elastic structures II: Numerical investigation." *Comp. Meth. Appl. Mech. Eng.*, Vol. **195**: **13-16**, 1458–1479, 2006.
- [8] J. Arbocz. "The Imperfection Data Bank, a Means to Obtain Realistic Buckling Loads." E. Ramm (Ed.) *Proceedings, Buckling of Shells – A State-of-the-Art Colloquium*, Springer Verlag, 535–567, 1982.
- [9] J. Arbocz and C. D. Jr. Babcock. "The Effect of General Imperfections on the Buckling Load of Cylindrical Shells." *J. Appl. Mech.*, Vol. **36**, 28–38, 1969.
- [10] W.T. Koiter. "Over de Stabiliteit van het Evenwicht." Dissertation, Delft. (1945) English translation: "On the stability of elastic equilibrium." *NASA TT F-10*, 1967.
- [11] R. Degenhardt, K. Rohwer, W. Wagner, J.P. Delseemme. "Postbuckling and collapse analysis of SFRP stringers stiffened panels – a GARTEUR activity." *Procs. Forth Int. Conf. Thin-Walled Str.*, Loughborough, UK, 2004.
- [12] W. Schneider. "Modelling of the Collapse Process of Quasi-Static Loaded Shell Structures." *Procs. of WCCM V*, Vienna, 2002.