

## Electrohydrodynamic deformation of vesicles and cells

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### ABSTRACT

We develop an analytical theory to describe the deformation of a closed lipid bilayer membrane (vesicle) in AC electric fields. The inner and suspending fluids, and the membrane, are treated as leaky dielectrics. The thin lipid membrane is impermeable to ions and thus it acts as a capacitor. The vesicle shape is obtained by evaluating electric and hydrodynamic stresses exerted on the membrane. Considering a nearly spherical shape, the solution to the electrohydrodynamic problem is obtained as a regular perturbation expansion in the excess area, which is the difference in the areas of the vesicle and an equivalent sphere with the same volume. The analysis also takes into account that the membrane is a flexible, fluid and incompressible shell. Interestingly, the constraint for a fixed total area leads to a non-linear shape evolution equation at leading order, which is independent of the membrane elastic properties.

The theory predicts that vesicle deformation in an AC field depends on field frequency, see Figure 1. If the inner fluid is more conducting than the suspending medium, vesicles are always prolate. In the opposite case, as the frequency increases, vesicles undergo a first shape transition from a prolate to oblate ellipsoid, and then from an oblate to prolate, in contrast to drops, which exhibit only the second transition. The theory shows that the frequency of the prolate-oblate transition,  $\omega_1$ , corresponds to the capacitor charging time, and therefore depends on the membrane thickness; this transition is non-existent for a membrane of zero thickness. The oblate-prolate transition depends solely on the electrical properties of the inner and outer fluid. Its critical frequency,  $\omega_2$ , is the same as in the case of a surfactant-covered drop as determined from electrohydrodynamic theory. The transition frequencies are independent of the viscosity contrast and membrane viscosity. The degree of vesicle deformation is limited by the excess area. The plateaus in Figure 1 correspond to a deformation where all excess area is transferred to a single ellipsoidal mode. The theory is in agreement with experimental data [1].

### REFERENCES

- [1] R. Dimova, K. Riske, S. Aranda, N. Bezlyepkina, R. Knorr, and R. Lipowsky. "Giant vesicles in electric fields" *Soft matter*, Vol. 3, 817–827, 2007.

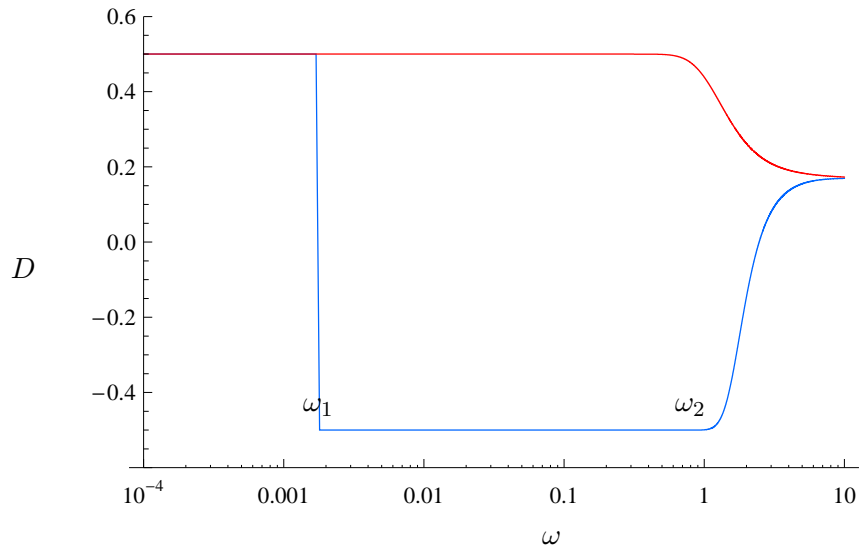


Figure 1: Ellipsoidal deformation,  $D = (r_{max} - r_{min}) / (r_{max} + r_{min})$ , for vesicles in a strong AC field ( $E_0 \sim kV/cm$ ) as a function of frequency. Red line is for conductivity ratio  $R = 1.5$  and blue line - for  $R = 0.5$ . The maximum value of the deformation corresponds to all excess area in the ellipsoidal mode, i.e.,  $D = \sqrt{\Delta/2}$ . In this case the excess area is  $\Delta = 0.5$ .  $D > 0$  means prolate deformation and  $D < 0$  -oblate. Frequency is non-dimensionalized by a charging time,  $t_c$ , given by the ratio of the dielectric constant and conductivity of the inner fluid.  $t_c \sim 10^{-7}s$