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BAYESIAN APPROACH FOR PARAMETER ESTIMATION IN HEAT TRANSFER

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ABSTRACT

A variety of techniques is nowadays available for the solution of inverse problems. However, one common approach relies on the minimization of an objective function that generally involves the squared difference between measured and estimated variables, like the least-squares norm, as well as some kind of regularization term.

Despite the fact that the minimization of the least-squares norm is indiscriminately used, it only yields *maximum likelihood* estimates if the following statistical hypotheses are valid: the errors in the measured variables are additive, uncorrelated, normally distributed, with zero mean and known constant standard-deviation; only the measured variables appearing in the objective function contain errors; and there is no prior information regarding the values and uncertainties of the unknown parameters. Although very popular and useful in many situations, the minimization of the least-squares norm is a non-Bayesian estimator. A Bayesian estimator is basically concerned with the analysis of the *posterior probability density*, which is the conditional probability of the parameters given the measurements, while the likelihood is the conditional probability of the measurements given the parameters.

If we assume the parameters and the measurement errors to be independent Gaussian random variables, with known means and covariance matrices, and that the measurement errors are additive, a closed form expression can be derived for the posterior probability density. In this case, the estimator that maximizes the posterior probability density can be recast in the form of a minimization problem involving the *maximum a posteriori objective function*.

On the other hand, if different *prior* probability densities are assumed for the parameters, the Posterior Probability Distribution does not allow an analytical treatment. In this case, Markov Chain Monte Carlo (MCMC) methods are used to draw samples of all possible parameters, so that inference on the posterior probability becomes inference on the samples.

In this work, we illustrate the use of Bayesian techniques for the estimation of parameters in heat transfer problems, via MCMC methods. Three algorithms are compared for the sampling procedure, namely: Metropolis-Hastings, Gibbs and a combination of them, as applied to a heat conduction problem.