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A MIXED MESHLESS FORMULATION FOR ANALYSIS OF SHELL-LIKE STRUCTURES

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ABSTRACT

In this contribution, a mixed meshless formulation, based on the Local Petrov-Galerkin approach proposed in [1], is developed for analysis of plate and shell structures. In contrast to the available displacement based meshless methods for analysis of shell-like structures [2,3,4], both the strains and displacements are approximated independently. The concept of a three dimensional (3-D) solid is used, allowing the application of complete 3-D constitutive models. Shell geometry is described exactly by employing a parametric mapping technique. Discretization of the shell continuum is carried out by couples of nodes located on the upper and lower surfaces. In order to obtain the governing equations, Petrov-Galerkin principle is applied over the local subdomain Ω_s^I surrounding the *I*-th couple of nodes, yielding the following local weak form of the 3-D equilibrium equations

$$\int_{\Omega_s^I} \left((C_{ijkl}\varepsilon_{kl})_{,j} + b_i \right) v_{ki} \,\mathrm{d}\Omega - \alpha \int_{\Gamma_{su}^I} (u_i - \bar{u}_i) v_{ki} \,\mathrm{d}\Gamma = 0; \quad I = 1,2,...,N \,. \tag{1}$$

Herein u_i and ε_{kl} are the displacement and strain components, respectively, which are considered as independent variables, and C_{ijkl} denotes the material tensor. b_i is the body force vector. The indices i, j, k, l take values 1, 2 and 3, while *N* stands for the number of couples of nodes used in the discretization. According to the Petrov-Galerkin approach, the test and trial functions may be chosen from different functional spaces. In this contribution, the test function written as $v_{ki} = \delta_{ki} v$ is assumed, where δ_{ki} is the Kronecker delta and v is the linear function through the thickness. The essential boundary conditions are enforced via penalty method with α as a penalty parameter, α>>1. Γ_{su}^I is the part of the local subdomain boundary Ω_s^I with prescribed displacements $\bar{u}_i$.

In order to eliminate the thickness locking effect, a new approach, using the interpolation of the transversal normal stress component, is developed. Herein, by means of the constitutive relation, the normal transversal strain component is replaced

by the normal transversal stress component. Thus, besides the displacement components, the independent variables are the five strain components and the transversal normal stress component as well. All variables are approximated by simple polynomial functions in the transversal directions and by means of the well-known Moving Least Square (MLS) interpolation in the in-plane directions. To obtain the discretized governing equations with only nodal displacements, the nodal strain and stress values are replaced by the displacement components using the collocation approach. Accordingly, the following additional relations are imposed in the classical weighted residual form, like in the formulation presented in [5]

$$\int_{\Omega_S^I} v_\varepsilon \left(\varepsilon_{\alpha\beta}^{(h)} - \varepsilon_{\alpha\beta}\right) \mathrm{d}\Omega = 0; \int_{\Omega_S^I} v_\varepsilon \left(\varepsilon_{\alpha 3}^{(h)} - \varepsilon_{\alpha 3}\right) \mathrm{d}\Omega = 0; \int_{\Omega_S^I} v_\sigma \left(\sigma_{33}^{(h)} - \sigma_{33}\right) \mathrm{d}\Omega = 0 . \qquad (2)$$

In this equation, $\varepsilon_{\alpha\beta}^{(h)}$ and $\varepsilon_{\alpha 3}^{(h)}$ are the interpolated in-plane strain components and transversal shear strain components, respectively, while $\sigma_{33}^{(h)}$ denotes the transversal normal stress interpolation. The other values in the brackets are calculated from the independently interpolated displacements. The values v_ε and v_σ are the Dirac's delta test functions at the nodes. By means of the procedure described, a closed system of equations is obtained on the global level with only the nodal displacements as unknown variables.

The proposed algorithm possesses considerable advantages in comparison to the standard fully displacement formulations. In a thin structural limit, the shear locking effect is fully alleviated even when low order MLS functions are used, contrary to the formulations presented in [2,3,4]. In addition, the required low order of the MLS interpolation enables the use of relatively small support domain, which significantly contributes to numerical efficiency. Differentiation of the MLS functions over the entire domain is avoided, which further increases the numerical efficiency in terms of computational costs and stability. The accuracy and robustness of the proposed formulation will be demonstrated by numerical examples.

REFERENCES

[1] S.N. Atluri, Z.D. Han, A.M. Rajendren, "A New Implementation of the Meshless Finite Volume Method, Through the MLPG "Mixed Approach"", *CMES: Comput. Model. Eng. Sci.*, Vol. **6(4)**, pp. 491-513, (2004).

[2] P. Krystl and T. Belytschko, "Analysis of Thin Shells by Element-Free Galerkin Method", *Int. J. Solids Structures*, Vol. **33,** No. 20-22, pp. 3057-3080, (1996).

[3] H. Noguchi, T. Kawashima, T. Miyamura, "Element free analyses of shell and spatial structures", *Int. J. Numer. Meth. Engng.*, Vol **47**, pp. 1215-1240, (2000).

[4] T. Jarak, J. Sorić, J. Hoster, "Analysis of shell deformation responses by the Meshless Local Petrov-Galerkin (MLPG) approach", *CMES: Comput. Model. Eng. Sci.*, Vol. **18**, pp. 235-246, (2007).

[5] T. Jarak and J. Sorić, "On Meshless Formulation for Modelling of Deformation Responses of Shell-Like Structures", *Proceedings of Special Workshop: ANASS 2007*, Zagreb, Croatia, pp. 95-113, 2007.