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Brittle Crack Growth driven by Material Forces in Functionally Graded Materials

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ABSTRACT

Functionally Graded Materials (FGMs) are advanced materials that possess continuously graded properties, see e.g.Surush and Mortensen [1]. Unlike to homogeneous materials, the propagation of cracks is strongly dependent on the gradation of the material. In this work a thermodynamic consistent framework for crack propagation in FGMs is presented. Following Miehe *et al.* [2] we exploit a global Clausius Planck inequality, where the direction of crack propagation is obtained in terms of material forces. Exploiting additional kinematical relations and a balance equation for equilibrium of forces finally the following coupled initial-boundary value problem is obtained, which accounts for both, evolution of deformation and evolution of crack propagation.

$$
\begin{array}{llll}
1. & \boldsymbol{\varepsilon} = \frac{1}{2}\left(\mathbf{h}+\mathbf{h}^T\right) & 2. & \mathbf{h} = \mathbf{u}\otimes\nabla \\
3. & \boldsymbol{\sigma}\cdot\nabla+\mathbf{b} = \mathbf{0} & 4. & \boldsymbol{\sigma} = \partial_{\boldsymbol{\varepsilon}}\psi \\
5. & \dot{\mathbf{A}} = \dot{\lambda}\dfrac{\mathbf{J}}{||\mathbf{J}||} & 6. & \Phi[\mathbf{J}] = ||\mathbf{J}||-\Gamma_c\leq 0 \\
7. & \dot{\lambda} \geq 0 \;,\quad \Phi[\mathbf{J}]\leq 0,\quad \dot{\lambda}\,\Phi[\mathbf{J}]=0 & 8. & \dot{\mathbf{u}} = \mathbf{v}-\mathbf{h}\cdot\mathbf{V} \\
9. & \mathbf{J} = -\lim_{\Gamma_\epsilon\to 0}\int_{\Gamma_\epsilon}\boldsymbol{\Sigma}\cdot\mathbf{n}\,\mathrm{dA} & 10. & \boldsymbol{\Sigma} = \psi\mathbf{1}-\mathbf{h}^T\cdot\boldsymbol{\sigma}
\end{array}
\tag{1}
$$

The final system of partial differential equations consist of the strain-displacement relation (1.1), where the displacement gradient is defined in (1.2). Eq.(1.3) expresses the balance equation and Eq. (1.4) is the elastic constitutive relation. The vector of crack propagation is defined in Eq.(1.5), and a Griffith-type crack criterion function is formulated in Eq.(1.6), along with the crack loading-unloading conditions (1.7). Eq.(1.8) relates the spatial velocity $\mathbf{v}$ to the material velocity $\mathbf{V}$. In Eq.(1.9) we have the definition of the material vector in terms of the Eshelby stress defined in Eq. (1.10). Additionally, appropriate Neumann and Dirichlet boundary conditions have to be formulated for the Cauchy stress tensor $\boldsymbol{\sigma}$ and the spatial and material velocity $\mathbf{v}$ and $\mathbf{V}$. Furthermore initial conditions $\mathbf{v}(t=0) = \mathbf{v}_0$, $\mathbf{V}(t=0) = \mathbf{V}_0$ are prescribed.

In the numerical implementation a staggered algorithm - deformation update for fixed geometry followed by geometry update for fixed deformation - is employed within each time increment. The geometry update is a result of the incremental crack propagation, which is driven by material forces. The corresponding mesh is generated by combining Delaunay triangulation with local mesh refinement. In order to improve the accuracy for the vectorial $J-$integrals a domain integral method is used, see [3].

Compact tension specimen with graded material: In the numerical example we consider a compact tension specimen with displacement control. The material gradation within the structure is expressed by a coordinate x_1^* with origin at the crack tip and which is rotated by an angle θ relative to the coordinate x_1. Then the corresponding Young's modulus is an exponential function of the form $E[x_1^*] = E_1 e^{\beta x_1^*}$. Two different angles for θ have been investigated: $\theta = 0^o$ and $\theta = 90^o$. For the case $\theta = 0^o$ the material is symmetric with respect to the y-axes, such that a crack starting from the notch propagating horizontally through the ct-specimen is expected theoretically. The results of both simulations are shown in Figure 1. From Figure 1, left, it is observed, that the algorithm captures the theoretical horizontal crack pattern very well. Furthermore the illustration in Figure 1 renders a strong upwards crack for gradation with angle θ = 90^o.

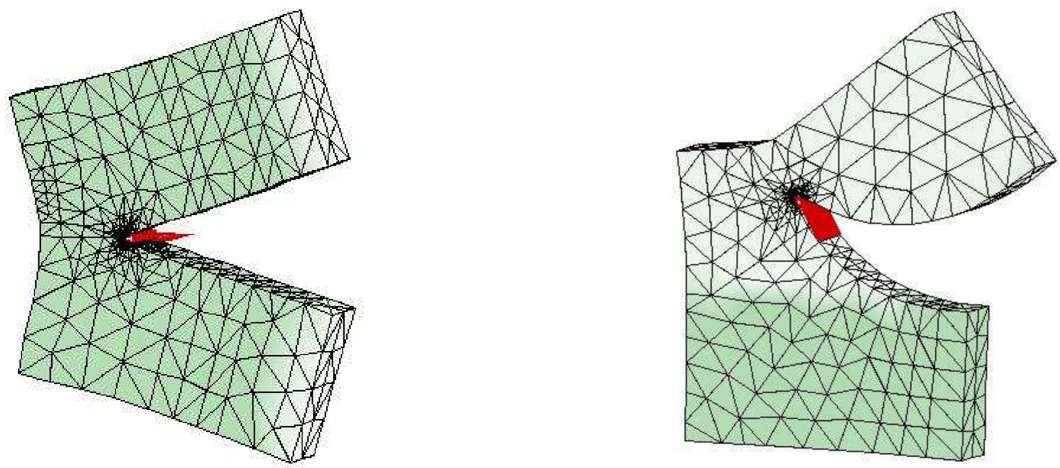

Figure 1: Compact tension specimen: crack propagation for θ = 0^o (left) and θ = 90^o (right).

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