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DYNAMIC ANALYSIS OF CRACKED PLATES REPAIRED WITH ADHESIVELY BONDED ANISOTROPIC PATCHES

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ABSTRACT

The aim of this paper is to present the formulation and matrix implementation of a dynamic analysis of cracked plates repaired with adhesively bonded composite patches, see Figure 1.

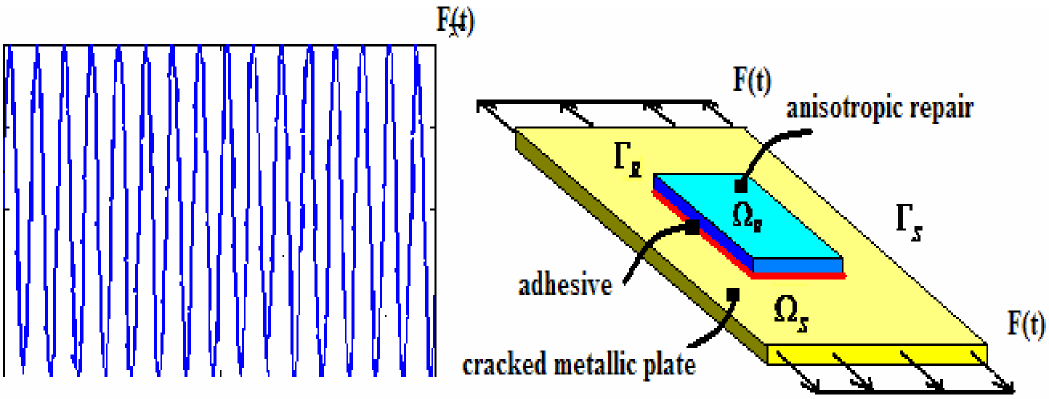

Figure 1: Cracked plate adhesively bonded with anisotropic patch

The numerical method that is used for modelling the problem of fracture mechanics is the dual boundary elements method (DBEM), which is possible to perform the modelling of a cracked plate in a single region. This method uses a displacement integral equation in one of the sides of the crack surface and a traction integral equation in the other side of the crack surface. The interaction effect between the plate and the anisotropic patch is modelled using the dual reciprocity boundary elements method (DRBEM), in order to analyse the shear stresses in the adhesive. Considering that the plate is under dynamic load, therefore adding a term to represent the inertial effects of each component in their respective equilibrium equations[1]. The integral equation for the plate (S) in a source point (x') is given by:

$$c_{ij}^{S}(x')\,u_j^{S}(x') + \int_{\Gamma_S} T_{ij}^{S}(x',x)\,u_j^{S}(x)\,d\Gamma = \int_{\Gamma_S} U_{ij}^{S}(x',x)\,t_j^{S}(x)\,d\Gamma + \frac{1}{h^S}\int_{\Omega_R} U_{ij}^{S}(x',x)\,b_j^{S}(x)\,d\Omega + \int_{\Omega_S} U_{ij}^{S}(x',x)\,\rho^S\,\ddot{u}_j^{S}(x)\,d\Omega \quad i,j=1,2 \tag{1}$$

Similarly, the integral equation for the repair (R) is given by:

$$c_{ij}^R(x')u_j^R(x')+\int_{\Gamma_R}T_{ij}^R(x',x)u_j^R(x)d\Gamma=\int_{\Gamma_R}U_{ij}^R(x',x)t_j^R(x)d\Gamma+$$
$$\frac{1}{h^R}\int_{\Omega_R}U_{ij}^R(x',x)b_j^R(x)d\Omega+\int_{\Omega_R}U_{ij}^R(x',x)\rho^R\ddot{u}_j^R(x)d\Omega\quad i,j=1,2 \tag{2}$$

Where $c_{ij}(x')$ is a coefficient that depends on the position of the source point (x') in relation to the boundary which is being integrated, $t_j(x)$ and $u_j(x)$ are the stresses and displacements of the system, $T_{ij}(x',x)$ and $U_{ij}(x',x)$ are the fundamental solutions for the tractions and displacements, h is the thickness of the component and ρ is the mass density of the component's material. The shear reactions in the adhesive $b_j(x')$ will be calculated by the difference between the displacements of the plate and the patch[1]:

$$b_j(x')=\frac{S_A}{h_A}\left\{u_j^S(x')-u_j^R(x')\right\}\quad j=1,2 \tag{3}$$

Where S_A is the shear module of the adhesive's material and h_A is the thickness of the adhesive layer. The effect of body forces, due to the masses of the plate and the patch under dynamic load, will be modelled by DRBEM. Discretizing the boundary of the system, equations for the plate and the repair can be written in a compact form as:

$$\begin{aligned}&\mathbf{H}_\Gamma^\mathbf{S}\,\mathbf{u}_\Gamma^\mathbf{S}-\mathbf{G}_\Gamma^\mathbf{S}\,\mathbf{t}_\Gamma^\mathbf{S}=\mathbf{A}_\Gamma^\mathbf{S}\,\boldsymbol{\alpha}^\mathbf{S}-\mathbf{B}_\Gamma^\mathbf{S}\,\boldsymbol{\rho}^\mathbf{S} \qquad &&\mathbf{H}_\Gamma^\mathbf{R}\,\mathbf{u}_\Gamma^\mathbf{R}-\mathbf{G}_\Gamma^\mathbf{R}\,\mathbf{t}_\Gamma^\mathbf{R}=\mathbf{A}_\Gamma^\mathbf{R}\,\boldsymbol{\alpha}^\mathbf{R}-\mathbf{B}_\Gamma^\mathbf{R}\,\boldsymbol{\rho}^\mathbf{R}\\ &\mathbf{u}_\Omega^\mathbf{S}-\mathbf{H}_\Omega^\mathbf{S}\,\mathbf{u}_\Omega^\mathbf{S}=\mathbf{A}_\Omega^\mathbf{S}\,\boldsymbol{\alpha}^\mathbf{S}-\mathbf{B}_\Omega^\mathbf{S}\,\boldsymbol{\rho}^\mathbf{S} &&\mathbf{u}_\Omega^\mathbf{R}-\mathbf{H}_\Omega^\mathbf{R}\,\mathbf{u}_\Omega^\mathbf{R}=\mathbf{A}_\Omega^\mathbf{R}\,\boldsymbol{\alpha}^\mathbf{R}-\mathbf{B}_\Omega^\mathbf{R}\,\boldsymbol{\rho}^\mathbf{R}\end{aligned} \tag{4}$$

Where $\mathbf{A}$ is given by $\mathbf{A}=\mathbf{H}^*\hat{\mathbf{U}}-\mathbf{G}^*\hat{\mathbf{T}}$ and $\mathbf{B}$ is given by $\mathbf{B}=\mathbf{H}\,\hat{\mathbf{U}}-\mathbf{G}\,\hat{\mathbf{T}}$, $\mathbf{H}^*$and $\mathbf{G}^*$ are similar to $\mathbf{H}$ and $\mathbf{G}$ but obtained by integration on the repair's boundary, α are unknown coefficients resulting from the use of the DRBEM technique, without physical meaning, and $\hat{\mathbf{T}}$ and $\hat{\mathbf{U}}$ are matrixes of tractions and displacements fundamental solutions. Making use of the DRBEM technique and the relation given by Eq.(3),it is possible to rewrite the term responsible for the interaction plate-repair as:

$$\mathbf{u}_\Omega^\mathbf{S}-\mathbf{u}^\mathbf{R}=\frac{h_A}{S_A}\mathbf{F}^\mathbf{S}\,\boldsymbol{\alpha}^\mathbf{S}\qquad \mathbf{u}^\mathbf{R}-\mathbf{u}_\Omega^\mathbf{S}=\frac{h_A}{S_A}\mathbf{F}^\mathbf{R}\,\boldsymbol{\alpha}^\mathbf{R} \tag{5}$$

Where $\mathbf{F}$ is the matrix of interpolation coefficients from the use of the DRBEM technique. Finally, coupling the equations for the plate and the repair using the DRBEM integration technique, the equation system, which rules the problem, is given by:

$$\begin{bmatrix}(\mathbf{H}-\mathbf{A}\,\mathbf{F}^{-1})^S & (\mathbf{A}\,\mathbf{F}^{-1})^S\\ (\mathbf{H}-\mathbf{A}\,\mathbf{F}^{-1})^R & (\mathbf{A}\,\mathbf{F}^{-1})^R\end{bmatrix}\begin{Bmatrix}\mathbf{u}^\mathbf{S}\\ \mathbf{u}^\mathbf{R}\end{Bmatrix}=\begin{Bmatrix}\mathbf{G}^\mathbf{S}\,\mathbf{t}^\mathbf{S}+\mathbf{B}^\mathbf{S}\,\boldsymbol{\rho}^\mathbf{S}\\ \mathbf{B}^\mathbf{R}\,\boldsymbol{\rho}^\mathbf{R}\end{Bmatrix} \tag{6}$$

REFERENCE

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