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Misfit functionals for recovering data in ElectroCardioGraphy problems

*** N. Hariga-Tlatli[1] and J. Erhel[2]**

[1] LAMSIN-ENIT & INAT
BP 37, 1002 Tunis Le Belvédère, Tunisia
tlatli.nejla@inat.agrinet.tn

[2] SAGE-IRISA.
Campus de Beaulieu Rennes, France.
Jocelyn.Erhel@irisa.fr

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ABSTRACT

The aim of this paper is to solve the inverse ElectroCardioGraphy (ECG) problem by the mean of an energy-like error functional and a generalized least-squares method.
The general framework of our contribution consists in recovering lacking data on some part of the boundary of a domain from overspecified data on the remaining part of the boundary. This kind of problem occurs in the reconstruction of cardiac activity. In fact, non invasive imaging of heart's electrical activity from ElectroCardioGram becomes a standard diagnostis tool in clinical application. The reconstruction of the spread of electrical excitation in the human heart of each single beat shall facilitate cardiologists to discriminate normal from abnormal activity, localize the origin of arrhythmias, ischemie or infarcted regions [5].
The approaches followed in this work have a common feature: they recover simultaneously both the unknown fields (potential and current). For the first one, designed by the generalized least-squares method at which Tikhonov-like regularizing term is added. The second one is an energy misfit functional. It is built on the H^1 semi-norm and turns out to be self regularizing is so far as it "filters" the highest frequencies.
The ECG problem is rephrazed as following:
Let Ω be the volume of the thorax, Γ_T the surface of the torso and Γ_H the surface of the heart so that they constitute a partition of the whole boundary $\partial\Omega$. Given a zero electrical flux, since the air around the body is insulating, and the corresponding potential T on the torso surface, one wants to recover the corresponding flux and potential on the heart surface. The most general setting takes into account both the non inhomogenities and the anistropy, in that case the conductivity is a variable matrix [4]. In our setting, we will consider an homogenious model problem:

$$\begin{cases} \nabla.\nabla u = 0 & \text{in} \quad \Omega, \\ \nabla u.n = 0 & \text{on} \quad \Gamma_T, \\ u = T & \text{on} \quad \Gamma_T. \end{cases} \tag{1}$$

This problem is known since Hadamard to be illposed in the sense that the dependence of u on the data T is not continuous [2]. We propose, in this paper, to reconstruct the lacking data using an energy-error functional introduced in [1] and the generalized least-squares method regularised by the Tikhonov procedure [3]. The two methods have in common the fact that they are based on two misfit functionals of least-squares type. The first one is built on the energy norm and the second one is a measurements to computations misfit function .

Computations have been run on FemLab for the error energy functional and MatLab for the generalized least-squares method.

To test the efficiency of the proposed reconstruction processes, we resort to synthetic data obtained from the numerical solving of the forward problem . We examine various ”simulated” measurements to test the capability of both the reconstruction methods and to compare their accuracy and robustness. The selected potentials correspond to data ranging from very smooth to severely singular one.

CONCLUSION

- As a general remark we would like to point out that both the considered recovering processes lead to a very satisfactory reconstruction of Dirichlet as well as Neumann boundary condition even for noisy data. However, as expected, better reconstruction is obtained by the two proposed methods when we deal with smooth data (i.e. data extendible to a harmonic function in hole the plane). Slight degradation is observed when one deals with singular fields. It appears from the experimental trials that the generalized least-squares method is much more sensitive to the noise then the energy method.
- The generalized least-squares method implemented with the boundary element is cheapear than the energy approach implemented with finite element method.
- At the light of the numerical study , it comes out that the two proposed missing boundary data recovering processes are pretty general provided that we have reliable computing tool to solve the corresponding forward problems.
- The pecular character of the method lies on the treatment of the reconstructed trace and norm derivative simoultaneously : both of them are well recovered.
- The generalized least-squares method seems to be more sensitive to noise however it is less expensive.
- The energy like method is, in our opinion, a relevent tool when treating inhomogeneities and anistropy.

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