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THE INTEGRATION OF THE CLASSICAL GEOMETRICALLY EXACT BEAM THEORY INTO THE ABSOLUTE NODAL COORDINATE FORMULATION

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ABSTRACT

The purpose of this study is to compare the absolute nodal coordinate formulation and the geometrically exact beam theory as proposed by Reissner [2] and implemented by Simo and Vu-Quoc [3] by means of the large rotation vector approach. The large rotation vector approach is widely used and has been extensively studied for two and three dimensional beam elements. Even though the strains of the shear deformable beam are deduced from local beam kinematical quantities rather than from the three dimensional continuum mechanics equilibrium equations, it can be shown that the formulation is objective. Finite elements based on the large rotation vector approach are discretized using position and rotational nodal coordinates and have been implemented in several commercial finite element codes. The discretization leads to a constant description of the mass matrix in the case of two dimensional elements. However, in three dimensional cases the mass matrix based on the large rotation vector approach cannot be constant, regardless of the choice of rotational coordinates. The absolute nodal coordinate formulation, on the other hand, is a recently developed approach that is inherently designed for multibody applications. In the formulation, the element is described by employing position and slope coordinates. The use of slope coordinates in the element transverse direction allows the formulation to account for the shear deformation as well as cross sectional deformation in the case of beams. Due to the use of slope coordinates, the shape function matrix with the vector of nodal coordinates can exactly describe large rigid body rotations. This leads to a constant description of the mass matrix feature which remains in effect also in the case of three dimensional elements. No rotational coordinates are employed in the absolute nodal coordinate formulation.

The absolute nodal coordinate formulation and the large rotation vector formulation differ considerably in the numerical formulation, although both approaches lead to a constant mass matrix in the planar beam formulation. It is noteworthy that mass

matrices are identical in both planar approaches when cross sectional deformations are neglected in the absolute nodal coordinate formulation. However, the original work of elastic forces in the shear deformable absolute nodal coordinate finite element, as proposed by Omar and Shabana [1], diverges from classical nonlinear rod theories. For this reason, two modifications to the original strain energy of the absolute nodal coordinate formulation are proposed in this study. The first modification is based on the St. Venant-Kirchhoff material. Previously, this material has been implemented into the absolute nodal coordinate formulation and it is identical with Reissner's shear deformable beam for the case of small deformations, but different in the large deformation case. In the second modification, the strain definitions, as proposed by Reissner [2] and implemented by Simo and Vu-Quoc [3], are applied to the absolute nodal coordinate formulation. Accordingly, this study shows that the large rotation vector and the modified absolute nodal coordinate formulation both lead to the same results in the case of large deformation and eigenvalue analysis. For this reason, it is appropriate to denote both approaches as 'geometrically exact', as no geometrical approximations are performed. The two dimensional absolute nodal coordinate beam element is further enhanced so that the convergence rate is quadratic in the mesh size for thin beams.

Numerical examples examined in this study include large deformation static and eigenvalue problems as well as the large deformation dynamic example of the well-known "flying spaghetti" problem [3]. The large rotation vector beam and the original as well as modified absolute nodal coordinate formulations are compared to analytical solutions in the case of static and eigenvalue analysis. All eigenmodes in the case of a simply-supported beam, approximated with the planar absolute nodal coordinate finite elements, provide analytical expressions. As an important result, it turns out that the eigenvalues of high-frequency modes caused by cross-section deformation are only slightly higher than the eigenvalues of shear-modes, independent from beam dimensions and Young's modulus. The high-frequencies, which can lead to numerical problems in some time integration methods, are thus approximately the same in the Simo and Vu-Quoc and the absolute nodal coordinate finite elements. Elastic forces based on a St. Venant-Kirchhoff material lead to slightly better agreement with the solution of planar solid finite elements for the case of eigenvalue and large deformation problems. Numerical results of the "flying spaghetti" problem are compared to a commercial finite element code.

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