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Some contributions to Perfectly Matched Layers

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ABSTRACT

Perfectly Matched Layers (PMLs) are a technique for simulating the absorption of waves in open domains. They have been introduced by Bérenger in 1994 [1] for the computation of time-dependent eletromagnetic waves and extended, since then, to other models of wave propagation, in the time domain but also for time-harmonic wave-like equations. They are an alternative to the use of Absorbing Boundary Conditions and became very popular due to the simplicity of their implementation and their efficiency. One of their main property is their perfect matching with the physical domain, for any angle of incidence and any frequency, at least for infinite layers.

The aim of this talk is to present several contributions we made to PMLs, mainly related to (i) the well-posedness and stability analysis of the continuous time-dependent PML system, (ii) the well-posedness and convergence of time-harmonic PML problems. We will conclude with some numerical questions (again related to the stability but on the discrete level) and open problems.

The question of well-posedness of time-dependent PML models has given rise to several works (e.g. [2], [3]...) and it is now clear that PML models (splitted and unsplit) are all (at least weakly) well-posed. But the notion of well-posedness is not sufficient in the sense that it does not prevent the solution from blowing up exponentially in time: in this case, we shall say that the problem is unstable. For Maxwell's equations, it has been proved (see [4]) that the "classical" split and unsplit PML models were well-posed and stable. The solution can then never be exponentially growing in time. However, it can be linearly growing at long time but this linear growth can be removed by using a new PML model, the PML-CFS (Complex Frequency Shift), see [5]. On the other hand, the classical PMLs can be unstable for some anisotropic models (elastodynamics, aero-acoutics...). In [3], it has been proved that there exists a necessary condition of stability related to the direction of the group velocity. In some cases, it is possible to design some new stable PMLs (e.g. [6-10]), but there are also cases where this question of stabilized PMLs is still open. Let us add that in [9], error estimates of PMLs of finite length are derived for the 2D time-dependent wave equation.

For time-harmonic problems, the convergence of the PMLs of finite length has been analyzed for Helmholtz equation in [11],[12], and for aero-acoustic problems in waveguides in [13],[14]. For these last anisotropic models, the presence of inverse upstream modes, which are responsable of the instability in the time domain, does not affect the convergence of PMLs in the frequency domain. This is not anymore the case when considering an elastic plate, as shown in [15]. In all these works, the convergence of PMLs of finite length depends on the length of the layer. In [16], the authors proposed a new PML model of finite length which is proved to be exact (for any length). We have extended this work to time-domain problems [17].

REFERENCES

[1] J. P. Bérenger. "A Perfectly Matched Layer for the Absorption of Electromagnetic Waves". *J. of Comp. Phys.*, Vol. **114**, 185–200, 1994.

[2] S. Abarbanel and D. Gottlieb. "A mathematical analysis of the PML method". *J. Comput. Phys*, Vol. **134**, No **2**, 357–363, 1997.

[3] E. Bécache, S. Fauqueux and P. Joly. "Stability of Perfectly Matched Layers, Group Velocities and Anisotropic Waves". *J. Comp. Phys.*, Vol. **188**, 399–433, 2003.

[4] E. Bécache and P. Joly. "On the analysis of Berenger's Perfectly Matched Layers for Maxwell's equations". *M2AN*, vol. **36**, No **1**, 87–120, 2002.

[5] E. Bécache, P. G. Petropoulos and S. Gedney. "On the long-time behavior of unsplit Perfectly Matched Layers". *IEEE AP-Transactions*, vol **52**, No **5**, May 2004.

[6] S. Abarbanel, D. Gottlieb and J. S. Hesthaven. "Well-Posed perfectly matched layers for advective acoustics". *J. Comput. Phys.*, vol. **154**, No **2**, 266–283, 1999.

[7] D. Appelö, T. Hagstrom, and G. Kreiss. "Perfectly Matched Layers for Hyperbolic Systems: General Formulation, Well-posedness, and Stability". *SIAP*, Vol. **67**, Issue **1**, 1–23, 2006.

[8] F. Q. Hu. "A stable, perfectly matched layer for linearized Euler equations in unsplit physical variables". *J. Comput. Phys.*, vol. **173**, 455–480, 2001.

[9] J. Diaz and P. Joly. "A time domain analysis of PML models in acoustics". *Computer Methods in Applied Mechanics and Engineering,*, vol. **195**, 3820–3853, 2006.

[10] F. Nataf. "A new approach to perfectly matched layers for the linearized Euler system". *J. Comput. Phys.*, Vol. **214**, No **2**, 757–772, 2006.

[11] M. Lassas and E. Somersalo. "On the existence and convergence of the solution of PML equations". *Computing*, vol **60**, 229–241, 1998.

[12] T. Hohage and F. Schmidt and L. Zschiedrich. "Solving time-harmonic scattering problems based on the pole condition: convergence of the PML method". *Konrad-Zuse-Zentrum für Informationstechnik Berlin* , No **01-23**, 2001.

[13] E. Bécache, A.-S. Bonnet and G. Legendre. "Perfectly matched layers for the convected Helmholtz equation". *SIAM Journal on Numerical Analysis*, Vol. **42**, No **1**, 409–433, 2004.

[14] E. Bécache, A.-S. Bonnet and G. Legendre. "Perfectly matched layers for time-harmonic acoustics in the presence of a uniform flow". *SINUM*, Vol. **44**, Issue **3**, 1191–1217, 2006.

[15] E. Bécache and A.-S. Bonnet. "PMLs for the numerical simulation of harmonic diffracted waves in an elastic plate". *World Congress on Ultrasonics*, Paris, 1019-1022, 2003.

[16] A. Bermúdez, L. Hervella-Nieto, A. Prieto, and R. Rodríguez. "An optimal perfectly matched layer with unbounded absorbing function for time-harmonic acoustic scattering problems". *J. Comput. Phys.*, Vol. 223, No **2**, 469–488, 2007.

[17] E. Bécache and A. Prieto. "PMLs with the singular σ in the time domain". *In preparation*.