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Application of a three-fields natural neighbour method in elastoplasticity

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ABSTRACT

The natural neighbour method can be considered as one of the many variants of the meshless methods. Classically, the development of these methods is based on the virtual work principle: a set of nodes are distributed over the domain to be studied and the displacement field is discretized with the help of interpolation functions that are not based on the finite element concept but only based on the nodes. In the present paper, we use a new approach based on the Fraeijs de Veubeke (FdV) functional [1] and initially developed for linear elasticity [2]. It uses separate discretizations of the displacements, stresses and strains. The method of [2] is extended to the case of geometrically linear but materially non linear solids and it is shown that, in the absence of body forces, the calculation of integrals over the area of the domain is avoided and that the derivatives of the nodal shape functions are not required, which constitutes an advantage over classical meshless and finite elements methods.

The Fraeijs de Veubeke functional for linear elasticity

For a 2D elastic solid A with a boundary $S = S_t \cup S_u$, the FdV functional writes:

$$\Pi(u_i, \varepsilon_{ij}, \Sigma_{ij}, r_i) =$$

$$\int_A W(\varepsilon_{ij})dA + \int_A \Sigma_{ij}\left[\frac{1}{2}(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}) - \varepsilon_{ij}\right]dA - \int_A F_i\, u_i\, dA - \int_{S_t} T_i\, u_i\, dS + \int_{S_u} r_i(\tilde{u}_i - u_i)\, dS \qquad (1)$$

where u_i is the displacement field, Σ_{ij} is the stress field, ε_{ij} is the strain field, $\tilde{u}_i$ are displacements imposed on the part S_u of the solid Boundary, r_i are the surface support reactions on S_u, T_i are the surface tractions imposed on S_t, F_i are the body forces and $W(\varepsilon_{ij})$ is the strain energy density.

In the natural neighbour method [3], the domain contains N nodes (including nodes on the domain contour) and the N Voronoi cells corresponding these nodes are built.
The discretization is based on those Voronoi cells and not on finite elements.

The following hypotheses are made: the assumed strains ε_{ij}, support reactions r_i and stresses Σ_{ij} are constant over each Voronoi cell while the assumed displacements u_i are interpolated by : $u_i = \sum_{J=1}^{N} \Phi_J u_i^J$ where Φ_J is a Laplace interpolant [4] and u_i^J is the displacement of node *J* corresponding to the Voronoi cell *J*.
Despite of the rich initial discretization, the method eventually leads to a system of equations of the classical type $[M]\{q\} = \{\tilde{Q}\}$ with $[M]$ a symmetric matrix, $\{q\}$ the set of unknown nodal displacements and $\{\tilde{Q}\}$ equivalent nodal forces.
A number of patch tests and other numerical examples show that the method allows solving problems involving nearly incompressible materials without locking.

Extension to non linear materials

In order to extend the previous method to materially non linear problems, we start from the following variational equation.

$$\delta\Pi(\dot{u}_i,\dot{\varepsilon}_{ij},\Sigma_{ij},r_i) = \int_A \sigma_{ij}\,\delta\dot{\varepsilon}_{ij}\,dA + \int_A \Sigma_{ij}\left[\frac{1}{2}\left(\frac{\partial\delta\dot{u}_i}{\partial x_j} + \frac{\partial\delta\dot{u}_j}{\partial x_i}\right) - \delta\dot{\varepsilon}_{ij}\right] dA + \int_A \delta\Sigma_{ij}\left[\frac{1}{2}\left(\frac{\partial\dot{u}_i}{\partial x_j} + \frac{\partial\dot{u}_i}{\partial x_j}\right) - \dot{\varepsilon}_{ij}\right] dA$$

$$- \int_A f_i\,\delta\dot{u}_i\,dA - \oint_{S_t} T_i\,\delta\dot{u}_i\,dS + \int_{S_u} \delta r_i(\dot{\tilde{u}}_i - \dot{u}_i)\,dS - \int_{S_u} r_i\,\delta\dot{u}_i\,dS = 0$$

σ_{ij} are the constitutive stresses at the considered material point.
These stresses are obtained by integration of a system of equations of the type:
$\dot{\sigma}_{ij} = f_{ij}(\sigma_{ij}, q_{ij}, \dot{\varepsilon}_{ij})$; $\dot{q}_{ij} = h_{ij}(\sigma_{ij}, q_{ij})$ with q_{ij} a set of internal variables.
The most important result of this development is that the 3 advantages mentioned in the linear case (no integration over the domain area, no need for the derivatives of the Laplace interpolant, final equation system of the classical type) are preserved throughout the iteration process. This is true no matter the non linear constitutive equation (elasto-plastic, elasto-visco-plastic, …) and no matter the time integration scheme used to integrate this constitutive equation. It remains also valid if the consistent tangent iteration matrix is computed analytically.

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