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HIGHER-ORDER LINKED INTERPOLATION FOR 3D BEAM ELEMENTS

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ABSTRACT

In this work a new family of linked interpolation functions for Timoshenko beam elements will be presented. Linked interpolation implies higher order interpolation for the transverse displacements than the rotations. It is well known that a polynomial interpolation of the same order used both for the transverse displacements and the rotations results in a phenomenon known as the shear locking [1]. This can be avoided by using the so-called reduced integration. However, this technique applies to the correction of the stiffness matrix and leaves unanswered the question of how exactly the displacement and rotation fields are interpolated.

We derive our linked interpolation by consistently providing internal degrees of freedom at a suitable number of internal nodes common for both the displacement and the rotation degrees of freedom, where this number depends on the order of the polynomial describing the applied loading. Exact solutions are obtained by solving the differential equation for such Timoshenko beam problem and expressing it in terms of the nodal values for the displacements and rotations. Thus, linked interpolation provides exact solutions for a general static polynomial loading and in this manner eliminates the problem of shear locking. This methodology is presented on a full 3D Timoshenko beam problem.

In this work we demonstrate how this approach gives exact solutions for 2, 3, 4 and 5-noded elements and that the solutions for a 2D case coincide with the known results [1-4]. There is a possibility for this methodology to be generalised to a higher-order Timoshenko beam elements of arbitrary order giving a family of interpolation functions which follow a very structured pattern. In particular, the linked interpolation derived here takes the following form for an element with n equidistant nodes [5,6]:

$$\boldsymbol{\theta} = \sum_{i=1}^{n} I^i \boldsymbol{\theta}_i \quad \text{and} \quad \mathbf{r} = \sum_{i=1}^{n} I_n^i \mathbf{r}_i - \frac{L}{n} \prod_{j=1}^{n} N_n^j \sum_{i=1}^{n} (-1)^{i-1} \binom{n-1}{i-1} \mathbf{G} \times \boldsymbol{\theta}_i,$$

where $\boldsymbol{\theta}$ is the interpolated rotation vector, $\mathbf{r}$ is the interpolated position vector, $\boldsymbol{\theta}_i$ and $\mathbf{r_i}$ are nodal rotation and position vectors respectively, I_n^i are the standard Lagrangian polynomials for the n-noded

beam finite element, $N_n^i = \frac{x}{L}$ for $i = 1$ and $N_n^i = 1 - \frac{n-1}{i-1}\frac{x}{L}$ otherwise. Here x denotes the arc-length co-ordinate of the beam, while $\mathbf{G}$ indicates the unit vector aligned with the undeformed centroidal axis of the beam.

The finite element with such interpolation provides exact solution provided the distributed loading is given by a polynomial function of order $n - 4$ or less, e.g. a three-noded element gives exact solution for concentrated loading, while a five-noded element gives exact solution for a trapezoidal loading. It should be stressed that by the exact solution here we mean not only the exact nodal values for the field variables, which is well known to be achievable by standard interpolation in conjunction with reduced integration, but also the exact field distribution and consequently the exact distribution of strain measures and stress resultants according to the Timoshenko beam theory.

The interpolation presented here has close links with the linearised version of the generalised interpolation given in [7] and can be naturally extended for application to higher-dimensional media with rotational degrees of freedom, e.g. plates.

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