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ON R-ADAPTIVE MESH OPTIMIZATION FOR LOCAL QUANTITIES OF INTEREST BASED ON THE MATERIAL RESIDUAL

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ABSTRACT

The error of a local quantity of interest depends on the error in the corresponding dual solution or generalized Green's function. This fact is used in dual-weighted based goal-oriented error estimation techniques and adaptivity algorithms. The physical residual is weighted with the error in the dual solution in order to obtain a suitable bound for the error in the quantity of interest. Furthermore, error indicators for mesh optimization are derived by weighting the error indicator of the primal problem with the corresponding one of the dual problem.

This contribution is concerned with goal-oriented r-adaptivity by using the so-called material residual. The material residual is also referred to as the weak form of the configurational force balance or pseudo-momentum equation and is usually used in the context of material inhomogeneities as well as any kinds of material defects, see e.g. the monographs [2, 3]. In the context of the finite element method, we obtain a material residual as a result of the non-optimal numerical solution, which is a result of the non-optimal discretization in the sense of the minimization of the overall energy [4, 5]. Hence, a non-vanishing material residual within the domain is an indicator for a non-optimal finite element mesh. This is used in different ways for global mesh optimization in recent years by some research groups, see the references in [4, 5]. The first discussion in the direction of configurational forces induced by finite element discretization was done by [1]. The mesh optimization problem has a long tradition. First steps for the optimization of finite element meshes based on discrete formulations of energy minimization goes back to the seventies, see e.g. [6]. In fact, the authors obtained the same discrete indicators for mesh optimization like the above mentioned approach from configurational mechanics, but they have not called them material or configurational forces.

A common approach for mesh optimization techniques is based on variational principles. We consider the total potential energy $E(\boldsymbol{u}, \boldsymbol{s})$ of a hyperelastic body. The energy depends on the state function $\boldsymbol{u} \in \mathcal{V}$ and on a *generalized design* or *control function* $\boldsymbol{s} \in \mathcal{D}$, which specifies in an abstract sense

the current reference configuration Ω_R, i.e. $\Omega_R = \Omega_R(\boldsymbol{s})$. In the context of the finite element method, the design function could be a subset of nodal coordinates which are allowed to vary and the reference configuration is the current mesh. If we use a geometrical representation, the design function could be a subset of control points of geometrical objects which specify the current reference configuration, e.g. Bézier curves. The space $\mathcal{V}$ denotes the usual Sobolev space and $\mathcal{D}$ the space with all admissible design functions. The partial variation of E with respect to $\boldsymbol{u}$ leads to the physical residual $\mathcal{R} : \mathcal{V} \to \mathbb{R}$. In the same manner, variation with respect to $\boldsymbol{s}$ leads to the material residual $\mathcal{G} : \mathcal{D} \to \mathbb{R}$. The overall minimization of the energy with respect to $\boldsymbol{u}$ and $\boldsymbol{s}$ requires the solution of the coupled physical and material problem. The simultaneous solution of the coupled problem ends in a system of linearized variational equations. Both problems are coupled by the *pseudo load operator*, which plays an important role for sensitivity analysis of physical and material quantities [4, 5]. Different numerical difficulties arise due to the simultaneous solution of this problem. Alternatively, a staggered solution method could be used, i.e. we solve the physical problem with a fixed design $\boldsymbol{s}$ and thereafter the material problem with a fixed deformation $\boldsymbol{u}$. For a given solution $\boldsymbol{u}$ of the physical residual $\mathcal{R}(\boldsymbol{u}, \boldsymbol{s}; \cdot)$, we have to solve $\mathcal{G}(\boldsymbol{u}, \boldsymbol{s}; \cdot) = 0$ within a Newton algorithm.

The resulting mesh is optimal with respect to the overall energy, but nothing could be said about the quality of local quantities of interest. We use the material residual of the primal and dual problem in order to obtain an error indicator for goal-oriented mesh optimization. The resulting mesh is optimal with respect to a local quantity of interest. By means of selected computational examples, we demonstrate the capability of the proposed framework.

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