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ON A MULTISCALE FRAMEWORK TO MODEL HETEROGENEOUS MATERIAL WITH STRONGLY COUPLED SCALES

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ABSTRACT

Beside the classical phenomenological approach [1], most of the multi-scale frameworks which mainly consist on the up-scaling of pertinent quantities from a finner scale, the micro scale, to a coarser one, the macro scale relies on a certain scale separation. From the classical homogenization analytical methods, such as bounds estimation [2], self-consistent model [3] or variational principle [4], that considers this scale separation as infinite, to the numerical homogenization technique based on domain decomposition and finite element method, such as the FE^2 method [5] that implicitly specifies a scale separation. Still, there are a lot of materials for which this scale separation does not hold, such as building material and among them concrete. For this class of material, we propose a multi-scale framework to handle strongly coupled scales, first introduced in [6].

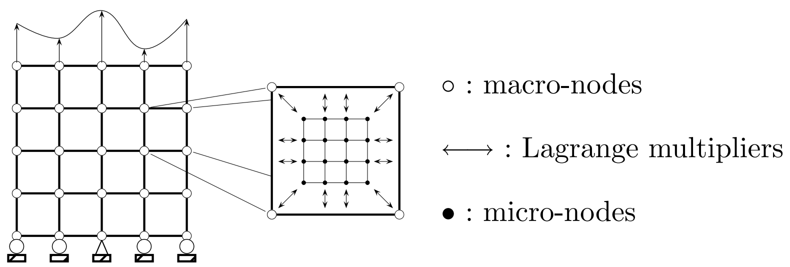

Fig. 1 – multi-scale problem

The presented method can be illustrated by Fig.1. Each element of the macro-scale coarse mesh contains a finner mesh that takes into account all the pertinent details of heterogeneous micro-structure. Each micro subdomain is hooked up to their macro-scale elements by localized Lagrange multipliers as in the algebraical FETI method [7]. From the total potential energy Π^{tot} (1),

$$\Pi^{tot} = \sum_{e=1}^{n_{el}} \Big(\underbrace{\int_{\Omega_e^m} (\Psi_e(\epsilon_e^m) - \bar{b}_e^m u_e^m)\, \mathrm{d}\Omega}_{\Pi_e^{int}(u_e^m)} + \underbrace{\int_{\partial\Omega_e^m} \lambda_e (u_e^m - u^M)\, \mathrm{d}S}_{\Pi_e^{\lambda}(u_e^m,\lambda_e,u^M)} \Big) - \underbrace{\int_{\partial\Omega_{vN}^M} u^M \bar{t}\, \mathrm{d}S}_{\Pi^{ext}(u^M)} \qquad (1)$$

it is possible to derive a variational formulation and from it, after several static condensations carried out with care because of the ill-posed micro-problems, it is possible to design a partitioned algorithm (see [8]). After a short presentation of the implementation in the context of the component technology using the CTL [9] of this multi-scale strategy, some results of numerical homogenization, both in linear and non-linear settings, will be discussed in this talk.

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