

Geometrically nonlinear 3D beam model based on Saint Venánt rod theory

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ABSTRACT

In order to recover geometrically nonlinear models, the implicit corotational formulation [1] is surely a very powerful tool. Its main feature is the possibility of obtain nonlinear models starting from linear ones in easy and automatic fashion. This is a great potentiality because the literature is very rich of linear theories and, conversely, the availability of nonlinear models is not completely satisfactory.

In particular, in the last two Centuries, a great effort has been spent in developing linear theory, leading to the classical Cauchy's theory of elastic tree-dimensional bodies, the Saint Venánt's rods theory, the plates and shells theories and, more recently, their numerical implementation using powerful computational approaches as the Finite Element Method. The so called "small displacements" assumption, allowing to refer to relatively simple linearized equations, have played a strong role in this evolution. A large amount of results obtained by the research, in particularly the derivation of structured models as plates and beams from the 3D continua and their description in terms of finite elements, are strictly related to this assumption.

However, the experience in deriving suitable nonlinear models is still poorer, particularly with respect to the so called *structural continua*, as beams, plates and shells, which are generally described through technical theories obtained by at-hoc heuristic assumptions intended to recover only 2nd-order accuracy. The need of a further research in this field is apparent. Our aim is to show that the implicit corotational approach, by decoupling the kinematical coherency from the elastic response, can provide an useful contribution in this direction and also allows to directly recover the results provided by the tradition.

The method is thereby applied in order to obtain a nonlinear 3D beam model able to accurately predict the effects due to out plane-warping of the cross section. In a well defined corotational frame the Saint Venánt rod theory [2,3], that gives a very accurate linear information in terms of stress and displacements, is fully reused. The obtained nonlinear beam model is able to describe very complex behavior, as for example Wagner effect[4]. A large amount of numerical analysis have been performed using Koiter's asymptotic approach [5]. The numerical and analytical results confirm the correctness and potential power of the proposed model.

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