

Elastic postbuckling analysis: how to recover accurate nonlinear structural models from the linear ones

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ABSTRACT

The asymptotic method, based on the implementation of the Koiter's approach to nonlinear instability within a finite element numerical context, represents a powerful tool for the analysis of geometrically nonlinear structures, which is able to provide accurate and reliable results through a fast computational scheme [1]. Because it make use of a fourth-order expansion of the energy, it however needs both the structural modeling and its finite element discretization to be also, at least, fourth-order accurate. This requirement is not satisfied by current nonlinear modelings for beams and plates which are generally only second order accurate, so their use can noticeably reduce the results accuracy. Conversely accurate (objective) models are too complex to be implemented in a finite element context or simply not allowable.

We will show that the implicit corotational formulation [2] can easily overpass this drawback by providing an automatic method for recovering accurate nonlinear models starting from the corresponding linear ones. The method is based on the polar decomposition theorem and the corotational description of motion [3], which is directly applied at the continuum level, following Biot [4]. The continuum body is thought of as subdivided in small regions, with a corotational reference system for each one that filter its average rigid motion. The accuracy increases with a reduction in the region size and the model become exact by a suitable limit process.

In the analysis of slender structures, undergoing large rotations and small strains, by referring to the linear stress solution as Biot's tensor in corotational frame and using a mixed variational formulation, we obtain an automated way of using the information gained by the linear analysis in the nonlinear context. Since linearized solution are always available even for complex beams, plates and shells, it is easy to obtain, by this way, the corresponding nonlinear models in a form convenient for numerical implementations.

The theoretical features and some tutorial implementation of the implicit corotational method are presented here regarding some implementation to planar and spatial beams and thin plates models. In these simple context, the methodology and the correctness of the proposed approach appear clearly. In particular for simplified 3D beam and thin plates we recover classical models already available in

literature [5]. Finally, the results of a numerical testing performed through both path-following and Koiter's asymptotic analysis are reported and compared, show the great accuracy provided by proposed approach.

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