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STABILITY OF ELASTIC PIPES CONVEYING FLUID WITH RIGID BODY DEGREES OF FREEDOM

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ABSTRACT

The structural stability of fluid conveying pipes has been subject to numerous investigations over the last decades, mainly due to their wide application in many different industrial fields. Results can be found, e.g. in the works by Païdoussis [1], applying Hamilton's principle for open systems or a Newtonian approach. In order to investigate the stability of pipes conveying fluid within the framework of computational codes for multibody systems, the equations of motion are derived using the extended Lagrange equations of motion for systems containing non-material volumes as presented by Irschik and Holl [2]. The kinetic and potential energies needed for setting up the Lagrange equations of motion are formulated using the Bernoulli-Euler beam theory in combination with the floating frame of reference formulation for rigid body rotations of the elastic pipe. In order to treat large deformations, the non-linear Lagrangian strain theory is utilized.

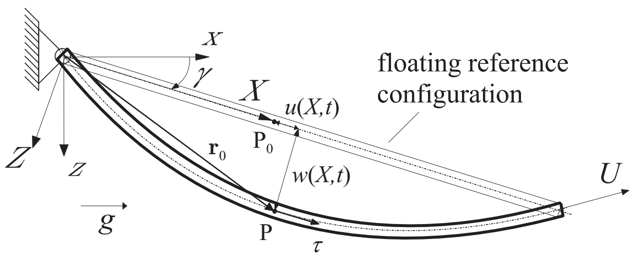

Figure 1: Elastic Pipe with Rigid Body Degree of Freedom

The position vector $\mathbf{r}_0^*$ of a point P_0 on the deformed pipe-axis and the tangent unit-vector $\boldsymbol{\tau}^*$, written in the floating frame of reference formulation (FFRF) for inextensible pipes, reads

$$\mathbf{r}_0^* = \mathbf{R}\mathbf{r}_0 = \left(X+u \quad w\right)^T, \boldsymbol{\tau}^* = \left\|\partial\mathbf{r}_0^* / \partial X\right\|^{-1}\left(\partial\mathbf{r}_0^* / \partial X\right) = \left(1+u' \quad w'\right)^T. \tag{1}$$

where X represents the spatial coordinate in the undeformed reference configuration, $u = u(X,t)$ and $w = w(X,t)$ are the axial and transverse displacements of the pipe-axis, respectively. The matrix $\mathbf{R}$ is a standard rotation matrix for the rigid body motion of the pipe, with the rigid body angle $\gamma = \gamma(t)$. The velocity vector $\mathbf{v}_{P0}^*$ of the pipe and of the fluid $\mathbf{v}_F^*$, written in the floating frame of reference formulation, follow with Eq. (1) to

$$\begin{aligned} \mathbf{v}_{P0}^* &= \mathbf{R}\left(\partial\left(\mathbf{R}^T\mathbf{r}_0^*\right)/\partial t\right) = \left(\dot{u} - \dot{\gamma}w \quad \dot{w} + \dot{\gamma}\left(X+u\right)\right)^T, \\ \mathbf{v}_F^* &= \mathbf{v}_{P0}^* + U\,\boldsymbol{\tau}^* = \left(\dot{u} - \dot{\gamma}w + U\left(1+u'\right) \quad \dot{w} + \dot{\gamma}\left(X+u\right) + Uw'\right)^T, \end{aligned} \tag{2}$$

where U denotes the scalar velocity of the fluid within the pipe. The equations of motion of the pipe system shown in Fig. 1 are deduced applying an extended form of the Lagrange equations of motion, written for systems containing non-material volumes, see Irschik and Holl [2], and follow to

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \int_S \mathrm{d}\mathbf{a} \cdot \left(\mathbf{v}_F - \mathbf{v}_{P0}\right) \frac{\partial T'}{\partial \dot{q}_i} - \int_S \mathrm{d}\mathbf{a} \cdot \left(\frac{\partial \mathbf{v}_F}{\partial \dot{q}_i} - \frac{\partial \mathbf{v}_{P0}}{\partial \dot{q}_i}\right) T' = Q_i. \quad (3)$$

The variables q_i and $\dot{q}_i$ denote a set of generalized coordinates and generalized velocities, respectively. Two surface integrals are included in the equations of motion, Eq. (3), in order to take into account mass transport. The surface integrals need to be evaluated at those parts of the boundary S, through which mass enters or exits the system. In the investigated pipe system the inlet and the outlet surfaces of the pipe contribute to these integrals only. The vector $\mathrm{d}\mathbf{a} = \boldsymbol{\tau} dA$ in Eq. (3) denotes an oriented surface element on S and A is the inner cross-sectional area of the pipe. The kinetic energy T in Eq. (3) consists of the kinetic energy of the pipe T_P and the kinetic energy of the fluid T_F and reads,

$$T_P = \frac{1}{2} m \int_0^L \mathbf{v}_{P0}^{*\,T} \mathbf{v}_{P0}^{*}\, dX, \quad T_F = \frac{1}{2} M \int_0^L \mathbf{v}_F^{*\,T} \mathbf{v}_F^{*}\, dX, \quad T = T_P + T_F. \quad (4)$$

The surface integral of the first kind in Eq. (3) needs to be evaluated at the inlet of the pipe only, since the surface integral vanishes at the inlet, and follows to

$$\begin{aligned} \int_{A_L} d\mathbf{a} \cdot \left(\mathbf{v}_F - \mathbf{v}_{P0}\right) \frac{\partial T'}{\partial \dot{q}_i} &= MU\left(\dot{u}_L - \dot{\gamma} w_L + U\left(1 + u_L'\right)\right)\left(\frac{\partial \dot{u}_L}{\partial \dot{q}_i} - w_L \frac{\partial \dot{\gamma}}{\partial \dot{q}_i}\right) \\ &+ MU\left(\dot{w}_L + \dot{\gamma}\left(L + u_L\right) + U w_L'\right)\left(\frac{\partial \dot{w}_L}{\partial \dot{q}_i} + \left(L + u_L\right)\frac{\partial \dot{\gamma}}{\partial \dot{q}_i}\right), \end{aligned} \quad (5)$$

where the index L denotes the outlet of the pipe and T' is the kinetic energy per unit-volume of the fluid.

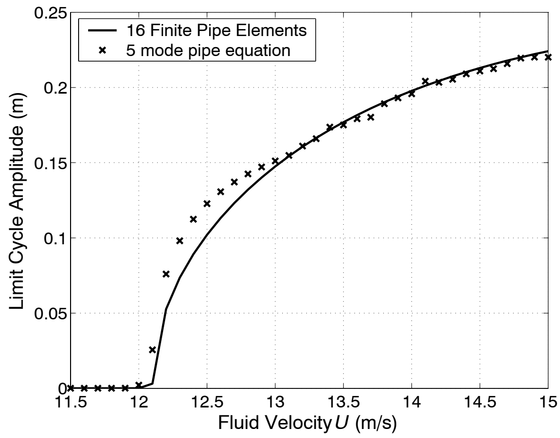

Figure 2: Limit Cycle Amplitude of the Transverse Deflection w

The results of a numerical stability investigation of the elastic pipe system with one rigid body degree of freedom, shown in Fig. 2, is compared to the solutions obtained using the multibody code HOTINT/MBS[1]. The pipe system is modelled in the multibody code using 16 finite pipe elements conveying fluid. The maximum amplitude of the transverse midspan deflection w with respect to the fluid velocity U is used for comparison. The stability boundary of the system can be observed at similar values of U around 12.1 m/s. Both results show good agreement over the range of investigated fluid velocities.

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[1] for details see: http://tmech.mechatronik.uni-linz.ac.at/staff/gerstmayr/hotint.html