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The *p*-version of the Finite Element Method

***Ernst Rank, Alexander Düster**

Chair for Computation in
Engineering, TU München,
Arcisstr. 21, 80333 München,
Germany
{rank,duester}@bv.tum.de

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ABSTRACT

Already very early Prof. Oleg Zienkiewicz pointed to potential advantages of using higher order instead of low order shape functions for the finite element method [1]. The *p*-version as a systematic extension process of the finite element method leaves the mesh unchanged and increases the polynomial degree of the shape functions locally or globally [5]. It has turned out to be a very efficient discretization strategy for many linear elliptic problems, for example, the Poisson equation, the Lamé equations, the Reissner–Mindlin problem, shell discretizations etc [e.g. 2,6]. For this class of problems the *p*-version is in general superior to the classical *h*-version approach. Also, in the case of singularities, the *p*-version shows an exponential rate of convergence in energy norm in the preasymptotic range, when combined with a proper mesh design [5].

In recent years, the *p*-version has been further developed to solve nonlinear problems such as hyperelasticity, elastoplasticity, fluid dynamics, fluid-structure interaction [e.g. 4,9] and it has been demonstrated that it can be efficiently applied to industrial problems arising for example in sheet metal forming [7]. A very important advantage of p-FEM is its ability to use elements with a very large aspect ratio. This feature allows to model even very thin-walled structures in a strictly three-dimensional setting [3]. Moreover, the inherent independence of the geometric shape from the space of the Ansatz functions opens many possibility to a thight connection of CAD-models and a p-FEM computation. In combination with an error estimation being based on the inherent hierarchic nature of the p-FEM, a model adaptivity for best approximation of a given physical problem is also possible [8].

The presentation will give an overview over the achievements of the *p*-version during the last two decades, addressing also recent results related to nonlinear problems in Computational Mechanics.

REFERENCES

[1] O.C. Zienkiewicz, B.M. Irons, J. Campbell, F.C. Scott. Three dimensional stress analysis. *Int. Un. Th. Appl. Mech. Symposium on High Speed Computing in*

Elasticity, Liege, 1970.

[2] E. Rank, R. Krause, K. Preusch. On the accuracy of p-version elements for the Reissner-Mindling plate problem. *International Journal for Numerical Methods in Engineering*, **43**:51-67, 1998.

[3] A. Düster, H. Bröker, E. Rank. The p-version of the finite element method for three-dimensional curved thin walled structures. *International Journal for Numerical Methods in Engineering*, **52**:673-703, 2001.

[4] A. Düster, A. Niggl, V. Nübel, E. Rank. A numerical investigation of high-order finite elements for problems of elasto-plasticity. *Journal of Scientific Computing*, **17**:429-437, 2002.

[5] B. Szabó, A. Düster, E. Rank, “The p-version of the Finite Element Method”, in E. Stein, R. de Borst, T.J.R. Hughes (eds.), *Encyclopedia of Computational Mechanics*, Vol. 1, pp. 119-139, John Wiley & Sons, (2004).

[6] E. Rank, A. Düster, V. Nübel, K. Preusch, O.T. Bruhns. High order finite elements for shells. *Computer Methods in Applied Mechanics and Engineering*, **194**:2494-2512, 2005.

[7] A. Muthler, A. Düster, W. Volk, M. Wagner, E. Rank. High order thin-walled solid finite elements applied to elastic spring-back computations. C*omputer Methods in Applied Mechanics and Engineering,* **195**:5377-5389, 2006.

[8] A. Düster, D. Scholz, E. Rank. pq-Adaptive solid finite elements for three-dimensional plates and shells. *Computer Methods in Applied Mechanics and Engineering,* **197**:243-254, 2007.

[9] U. Heißerer, S. Hartmann, A. Düster, W. Bier, Z. Yosibash, E. Rank. p-FEM for finite deformation powder compaction. *Computer Methods in Applied Mechanics and Engineering,* **197**:727-740, 2008.