# VORTEX STRUCTURES AROUND A SPHERE MOVING IN THE STRATIFIED FLUID 

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#### Abstract

At the present paper the stratified viscous fluid flows around a sphere are investigated by means of the direct numerical simulation (DNS) on the massively parallel computers with a distributed memory and the $\beta$-visualization of the 3D vortex structures at the following ranges of the internal Froude Fr and Reynolds Re numbers: $0.004 \leq F r \leq 10,10 \leq R e \leq 1000$ ( $F r=U /(N \cdot d), R e=U \cdot d / v$; where $d$ is a sphere diameter, $N$ is a buoyancy frequency).


For solving of the Navier-Stokes equations in the Boussinesq approximation (including the diffusion equation for the stratified component (salt)) the Splitting on physical factors Method for Incompressible Fluid flows (SMIF) with the hybrid explicit finite difference scheme (second-order accuracy in space, minimum scheme viscosity and dispersion, monotonous) has been used [1]. The Poisson equation for the pressure has been solved by the Preconditioned Conjugate Gradients Method.

For the visualization of the 3D vortex structures in the sphere wake the isosurfaces of $\beta$ have been drawn, where $\beta$ is the imaginary part of the complex-conjugate eigen-values of the velocity gradient tensor $\mathbf{G}$ [2]. $\beta$ has a real physical meaning. Let us consider a local stream lines pattern around any point in a flow (where $\beta>0$ ) in a reference frame $\mathbf{x}$ moving with the velocity of this point $(\mathbf{v}=\mathrm{d} \mathbf{x} / \mathrm{d} t \approx \mathbf{G} \mathbf{x}$, where $\mathbf{v}$ is a velocity of a fluid particle in the considered reference frame $\mathbf{x}$ and $t$ is time). It's easy to demonstrate (see the theory of the ordinary differential equations) that the local stream lines pattern in the considered reference frame $\mathbf{x}$ is closed or spiral, and $\beta$ is the angular velocity of this spiral motion. The good efficiency of this $\beta$-visualization technique has been demonstrated in [3].

Previously the homogeneous fluid flows $(F r=\infty)$ with one non-dimensional parameter Re have been investigated. With increasing of Re the number of the degrees of freedom is increased too. The continuous changing of the wake vortex structure is observed with increasing of Re [3]. The detailed formation mechanisms of vortices (FMV) in the sphere wake at $200 \leq R e \leq 1000$ have been described in [3] (for the first time). In particular it was shown that the detailed FMV for $270<R e \leq 290,290<R e \leq 320$ and $320<R e \leq 400$ are
different. (The main differences have been observed in the recirculation zone.)
In the case of the stratified fluid the full 3D vortex structure of the flow around a sphere is not clear for many flow regimes [4]. In addition the clear understanding of the continuous changing of the full 3D sphere wake vortex structure with decreasing of Fr is also absent. At the present paper this continuous changing of the wake structure is investigated in detail at $R e=100$ (for the first time) by using DNS and the $\beta$-visualization technique. At $F r>10$ the fluid is practically homogeneous. At $0.03 \leq F r \leq 3$ the lee waves with length $\lambda / d \approx 2 \pi \cdot F r$ (in the vertical plane) are observed (Fig. 1). At $0.9<\mathrm{Fr}<10$ the non-axisymmetric attached vortex is observed in the recirculation zone. At $\mathrm{Fr} \approx 0.9$ the quasi-rectangular vortex in the recirculation zone is transformed into two symmetric vortex loops. At Fr $\approx 0.6$ the four legs of these vortex loops are connected with the corresponding vortex threads (Fig. 1b) induced in the wake along the sphere trajectory; the flow inside these legs are redirected downstream; the vortex loops are shifted from the sphere surface and the primary separation line on the sphere surface vanish (Fig. 1c). At $\mathrm{Fr} \approx 0.4$ a new recirculation zone is formed from the "wave crest" which is situated very close to the sphere. At $\mathrm{Fr}<0.25$ two steady vertical vortices (bounded by lee waves) are attached to the sphere. The obtained horizontal and vertical separation angles are in a good agreement with the experiment [4].
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Fig. 1. The vortex structure of the sphere wakes at $R e=100$ : a) $F r=2(\beta=0.055)$, b) $F r=1(\beta=0.02)$, c) $F r=0.5(\beta=0.02)$.

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