8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

ADER-DG methods for geophysical aplications

*** Cristóbal E. Castro[1], Josep de la Puente[1] and Martin Käser[1]**

[1] Ludwig-Maximilians-Universität München, Earth and Environmental Sciences (Geophysics)
Theresienstr. 41, 80333 München, Germany
castro@geophysik.uni-muenchen.de http://www.geophysik.uni-muenchen.de/Members/castro

Key Words: *ADER-DG, seismic wave, shallow water, unstructured meshes, high order.*

ABSTRACT

Computational methods for wave propagation found application on several physical areas of interest. We are concerned with numerical method for solving hyperbolic equations with arbitrary high order of accuracy in space and time while general enough to be used on different equations without introducing any ad-hoc procedure.

We use the ADER-DG [1] method which introduces spectral convergence using spatial polynomial representation of the data and high order time integration due to the ADER [2], [3] approach. Unstructured meshes are used in 2 and 3 space dimensions for linear and nonlinear hyperbolic balance laws.

The spatial polynomial representation of the data is obtained with a linear combination of basis functions multiplied with time-dependent coefficients called degrees of freedoms. There is more than one choice of these basis functions when quadrangular meshes are used, for example modal basis and nodal basis. We compare these basis functions for quadrangular and hexahedral meshes as well the combination of regular quadrangular and hexahedral meshes with triangular and tetrahedral meshes.

These basis functions are defined on reference elements where many spatial integrals can be performed at initial time and then used during computational time. We can evaluate the polynomial data on the physical or reference space coordinate using the corresponding mapping. This mapping can take into account the deformation of the element and curvilinear boundaries.

In spectral element method the nodal basis functions are used ensuring continuity between two neighbor elements. This condition is not required by ADER-DG methods allowing the use of the modal basis and introducing the concept of flux computation extensively used in the finite volume community.

High order computation of the numerical flux needs to solve the Derivative Riemann Problem [4] which is an initial value problem with discontinuous polynomial initial data, taking into account the possible source term.

We apply this method to the linear elastic wave equation; see for example [5], [6], [7] and [8], and the linear and non linear shallow water equation [9].

REFERENCES

[1] M. Dumbser. “Arbitrary High Order Schemes for the Solution of Hyperbolic Conservation Laws in Complex Domains”. PhD thesis, Universität Stuttgart, Institut für Aerodynamik und Gasdynamik, 2005.

[2] E. F. Toro, R. C. Millington, and L. A. M. Nejad. “Towards very high order godunov schemes”. In E. F. Toro, editor, *Godunov methods; Theory and applications*, pages 907–940, Oxford, 2001. Kluwer Academic Plenum Publishers

[3] V. A. Titarev and E. F. Toro. “ADER: Arbitrary high order Godunov approach”. *J. Sci. Comput.*, Vol. **17**, 609–618, 2002.

[4] C.E. Castro and E.F. Toro. “Solvers for the high-order riemann problem for hyperbolic balance laws”. *J. Comput. Phys. (in Press)*, Vol. **–**, – , 2007.

[5] M. Käser and M. Dumbser. “An arbitrary high order discontinuous Galerkin method for elastic waves on unstructured meshes I: The two-dimensional isotropic case with external source terms”. *Geophys. J. Int.*, Vol. **166**(2), 855–877, 2006.

[6] M. Dumbser and M. Käser. “An arbitrary high order discontinuous Galerkin method for elastic waves on unstructured meshes II: The three-dimensional case”. *Geophys. J. Int.*, Vol. **167**(1), 319–336, 2006.

[7] M. Käser, M. Dumbser, J. de la Puente, and H. Igel. “An arbitrary high order discontinuous Galerkin method for elastic waves on unstructured meshes III: Viscoelastic attenuation”. *Geophys. J. Int.*, Vol. **168**(1), 224–242, 2007.

[8] J. de la Puente, M. Käser, M. Dumbser, and H. Igel. “An arbitrary high order discontinuous Galerkin method for elastic waves on unstructured meshes IV: Anisotropy”. *Geophys. J. Int.*, Vol. **169**(3):1210–1228, 2007.

[9] C.E. Castro. “High order ADER FV/DG numerical methods for hyperbolic equations”. Monographs of the school of doctoral studies in environmental engineering, University of Trento, Civil and Environmental Engineering, 2007.