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A SUBGRID DISCONTINUOUS GALERKIN METHOD FOR ADVECTION-DIFFUSION-REACTION PROBLEMS

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ABSTRACT

A discontinuous subgrid scale method for the numerical solution of advection-diffusion-reaction equations is proposed in this work. It reformulates the nonlinear subgrid scale (NSGS) finite element model developed in [3, 4] using broken spaces. The original NSGS method is based on a multiscale (two-level) approach in which a nonlinear artificial viscosity term is added only to the subgrid scales of a finite element mesh. The amount of the subgrid viscosity is scaled by the resolved scale solution at element level, yielding a self adaptive method. The Galerkin method is recovered when the resolved scale solution is accurate enough.

Following this approach, the approximation space X_h is split into resolved scales (X_H) and subgrid scales (X_h^H) spaces. The multiscale discrete setting is composed by two nested meshes such that the coarsest is a triangular partition $\mathcal{T}_H = \{T_H\}$ of the domain. From each triangle $T_H \in \mathcal{T}_H$, four triangles are created by connecting the midpoints edges. We denote by $\mathcal{T}_h = \{T_h\}$ the resulting finer triangulation. The space X_h is required to be continuous for each $T_h \in T_H$ although it is discontinuous along the meshlines of $\mathcal{T}_H$. Thus, a two-level piecewise linear finite element approximation is adopted so that the subgrid scale solution may be nonzero across the $\mathcal{T}_H$ meshlines. The new subgrid discontinuous method considers the residual of the approximation inside each element and its jumps across interelement boundaries, taking into account the subgrid terms.

Numerical experiments are conducted to cover a variety of parameter ranges in order to show the behavior of the proposed methodology in comparison with some discontinuous Galerkin methods [1, 2].

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