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OBJECT ORIENTED HP ADAPTIVE FINITE ELEMENT METHOD SYSTEM FOR MULTISCALE PROBLEMS

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ABSTRACT

We present a detailed object-oriented project of the mutli-dimensional hp adaptive Finite Element Method (FEM) system dedicated for multi-scale problems. The project is described by utilizing the Unified Modelling Language (UML) [1].

The system is designed to support computational meshes of an arbitrary spatial dimension. It is done by utilizing template classes with parameter *N* denoting the spatial dimension, and the recursive definition of a multi-dimensional cubic finite element, following [2, 3]. The 1D finite element consists in two vertex nodes and one edge node. The vertex node local shape functions are first order polynomials, whilst edge node local shape functions are of the arbitrary higher order.

The local shape functions over higher dimension finite elements are defined by tensor products of 1D local shape functions. The *n*-dimensional vertex node shape functions are defined by tensor product of *n* 1D vertex shape functions, the *n*-dimensional edge node shape functions are defined by tensor product of one 1D edge shape functions and *n-1* 1D vertex shape functions. All *n*-dimensional shape functions are constructed by all possible permutations of 1D vertex and edge shape functions, compare Fig. 1.

The global shape functions, utilized to identify rows and column of the stiffness matrix, are collections of suitable local shape functions, identified by their geometrical location.

The computational mesh can either be *h* or *p* refined. The *h* refinement consists in the construction of some new finite element nodes. For exampe, breaking a 1D element edge means to create one new vertex node and two new edge nodes. Following [3, 4] the *h* refinements are expressed as trees of nodes growing from the initial mesh elements. This is expressed by father / sons relation between nodes on the UML diagram presented in Fig. 1. The *p* refinement is expressed just by adding new polynomial orders of approximation to selected geometrical object.

The list of active elements is constructed dynamically, based on the element nodal connectivities. The system supports arbitrary elliptic problem variational formulation.

For the multi-scale computations, the equations that are solved over a finite element, depends on the element dimensions. The equations can be changed from the macro scale problem into the nano or micro scale problems. The suitable interface conditions are implemented on the interface between two scales.

The presentation will be enriched by the preliminary numerical results of the 1D linear elasticity multi-scale problem.

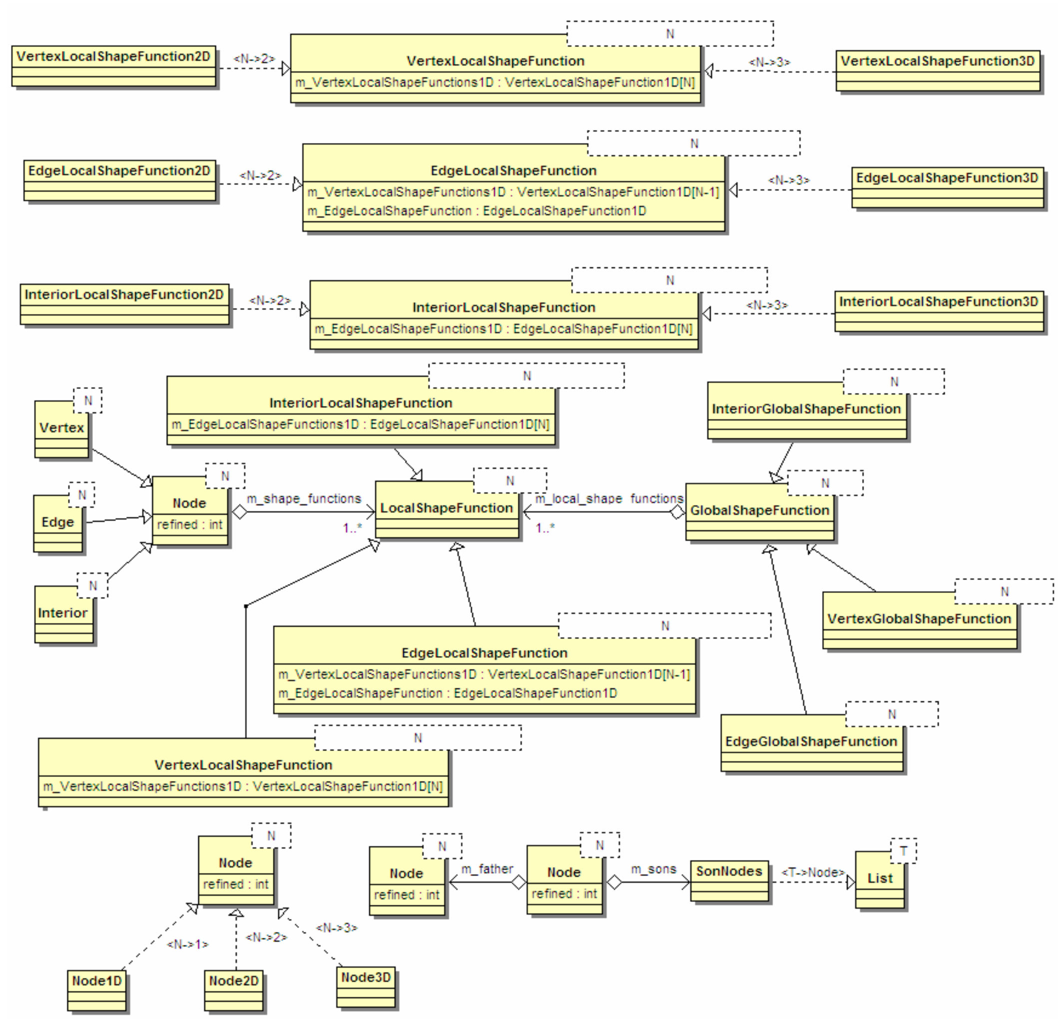

Fig. 1 UML diagrams expressing nodal connectivities and shape functions relations.

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