8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering
(ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

High-Order Upwind and WENOM Schemes for DNS of Compressible Turbulent Flow

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Key Words: *DNS, Compressible Wall-Turbulence, Upwind Schemes, WENOM Reconstruction.*

ABSTRACT

The present paper studies high-order upwind and WENOM reconstructions [1, 2] for the DNS of compressible wall-bounded turbulent flows [3]. These reconstructions are coupled with an HLLC [4] approximate-Riemann-solver (ARS). Most non-spectral compressible DNS solvers use upwind schemes, with WENO-type reconstructions [5, 6] up to $O(\Delta x_{\mathrm{H}}^{7})$. Balsara and Shu [1] have developed WENO schemes up to $O(\Delta x_{\mathrm{H}}^{11})$, and Henrick et al. [2] have improved the accuracy of the WENO5 scheme, by introducing the mapped-WENO scheme (WENOM5).

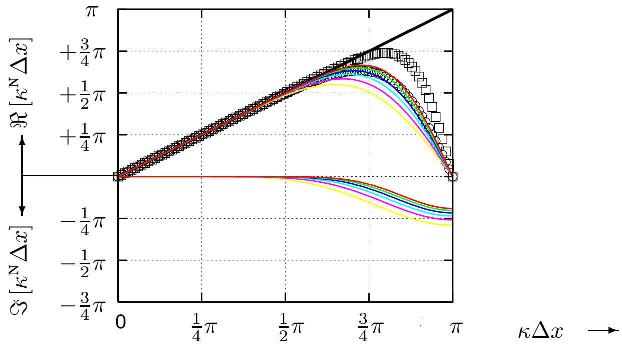

Figure 1: Comparison of modified-wavenumber for the derivative of a function [7] using the UW07, UW09, UW11, UW13, UW15, and UW17 upwind schemes [1, 8], the CCP06 and CCP10 compact centered Padé schemes of Lele [7] (3-diagonal (CCP06) and 7-diagonal CCP10).

In the present work we develop WENO and WENOM schemes up to $O(\Delta x_{\mathrm{H}}^{17})$ ($r = 9$). The UW17 scheme (WENOM17 scheme with optimal weights) has equivalent spectral resolution (Fig. 1) with the $O(\Delta x_{\mathrm{H}}^{6})$ central compact tridiagonal Padé (Hermitian) scheme (CCP6) of Lele [7]. These schemes are evaluated, against standard test-problems for both the biconvection equation and the 1-D and 2-D Euler equations [8, 9].

Then they are applied to the DNS computation of compressible channel flow [3, 10]. Simple correlations (such as second-order-moments of flow variables) are well predicted by all of the schemes. On the contrary (Fig. 2) more difficult correlations, such as some third-order-moments or some second-order-moments of velocity-gradients (*eg* $\overline{w'w'v'}$, ϕ_{xy} or ε_{yy}) establish a clear hierarchy in the performance of the different schemes, on a given grid, consistent with the spectral resolution properties

(Fig. 2). On the other hand, the use of nonlinear WENO weights strongly reduces the performance of the scheme, *eg* the WENO11 scheme gives results equivalent to the linear UW5 scheme. Interestingly, using WENOM nonlinear reconstruction substantially improves performance, the WENOM11 scheme giving results close to the UW7 linear scheme. Results with the WENOM17 scheme ($r = 9$) are presented in [9].

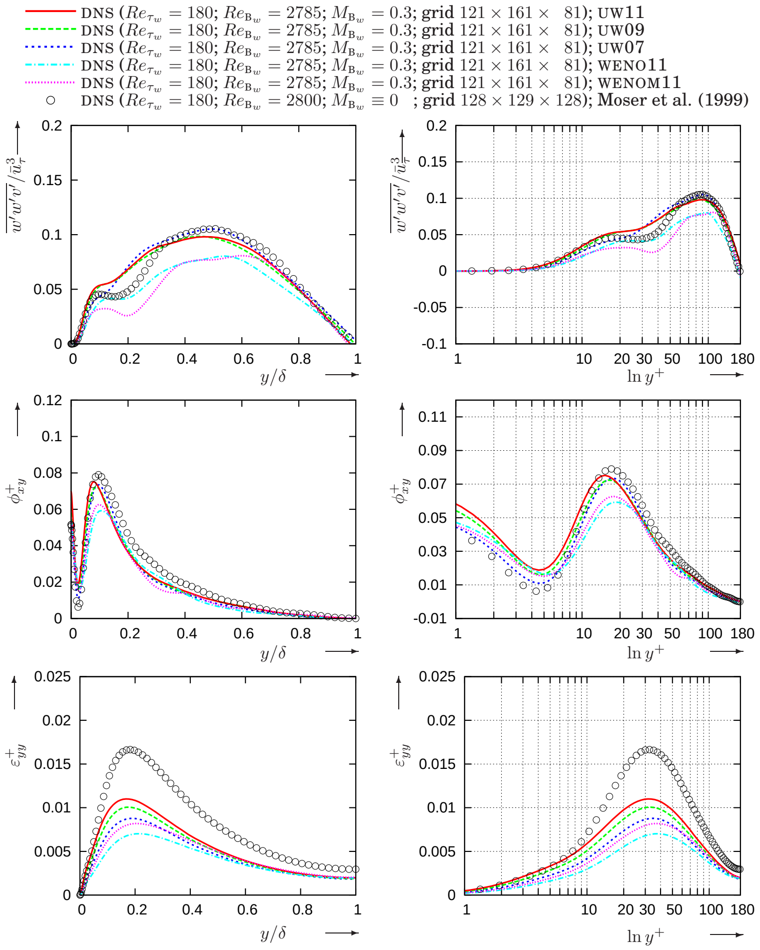

Figure 2: Comparison of present DNS-computed statistics ($Re_{\tau_w} = 180$; $M_{B_w} = 0.3$; $\bar{M}_{CL} = 0.34$), with the UW7 ($t^+_{OBS} \cong 2000$), UW9 ($t^+_{OBS} = 3000$), UW11 ($t^+_{OBS} = 2500$), WENO11 ($t^+_{OBS} = 1000$), and WENOM11 ($t^+_{OBS} = 700$) schemes (grid $121 \times 161 \times 81$), with incompressible DNS results of Moser et al. [10] ($Re_{\tau_w} = 180$; $M_B \equiv 0$).

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