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A COUPLED MODEL FOR TRANSPORT PROCESSES AND DEFORMATIONS IN LANDFILLS

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ABSTRACT

Municipal solid waste landfills are constructed to safely separate the disposed waste from the environment. After emplacement, the waste undergoes a characteristic time-settlement curve [1]. Initial deformations are generated by the compressive dead loads of overburden lifts. Highly deformable particles exhibit a viscous behaviour, which produces settlement on the macroscopic level within few days. In addition, consolidation is connected to the transport of pore liquids existing in micropores of agglomerations of solid particles as well as in the larger crevices between such agglomerations. Against it, the long-term behaviour is governed by the biodegradation of organic substances. Carbohydrates, fats, proteins, etc. are decomposed by microorganisms into gas, methane and carbon dioxide mainly, water and acids. The different processes of solid deformation, of fluid transport and of biodegradation interact strongly. In this contribution, a model approach is discussed, which describes the coupled processes of transport and deformation only, whereas degradation is neglected totally.

Municipal solid waste is treated as a porous medium. Thereby, voids between single solid particles are partially filled by liquid and by gas. According to the concepts of the *Mixture Theory*, e.g. Bowen [2], and the *Theory of Porous Media*, see de Boer [3], each of the constituents - solid, gas and liquid - exists at all material points of the body under consideration at the same time. The presence of a constituent in a representative elementary volume is described by the volume ratios, which change due to the deformation and transport processes. For all constituents as well as for whole mixture, the general principles of conservation of mass, linear momentum, angular momentum and energy are discussed in detail in the literature. Assuming isothermal conditions, neglecting all inertial forces and applying Darcy relation of fluid transport for the unsaturated case, the mass balances of the two liquids and the balance of linear momentum of the mixture are the coupled partial differential equations governing the investigated problem.

Splitting the velocity of a fluid $\mathbf{v}^f$ into the velocity of the solid $\mathbf{v}^s$ and the its seepage velocity $\mathbf{w}^f$, which is related to the fluid pressure by the Darcy relation, the fluid mass balance states

$$\frac{\mathrm{d}^s\, \Phi\, S^f\, \tilde{\varrho}^f}{\mathrm{d}\, T} + \Phi\, S^f\, \tilde{\varrho}^f\, \mathrm{div}\,(\mathbf{v}^s) + \mathrm{div}\left(\Phi\, S^f\, \tilde{\varrho}^f\, \mathbf{w}^f\right) = 0\,. \tag{1}$$

Thereby, the physical density $\tilde{\varrho}^f$ is related to the fluid pressure by the ideal gas law for the gaseous constituent, whereas incompressible behaviour is assumed for the liquid. The porosity Φ depends on the solid deformation. The saturation S^f is altered by the transport and the solid deformation.

The balance of linear momentum for the whole mixture ensures force equilibrium

$$\operatorname{div}\left(\mathbf{T}^s_{eff} - p\,\mathbf{1}\right) + \left(\varrho^s + \Phi\, S^l\, \tilde{\varrho}^l + \Phi\, S^g\, \tilde{\varrho}^g\right)\,\mathbf{b} = \mathbf{0}\,. \tag{2}$$

The total CAUCHY stresses are splitted into the effective stresses $\mathbf{T}^s_{eff}$ carried by the solid skeleton and the pore pressure p formed by the gas and liquid pressure, respectively. Net forces originate from the weight of the different constituents. Displacements and fluid pressures are prescribed on DIRICHLET boundary, whereas fluid fluxes and surface tractions act on von NEUMANN boundaries.

The large deformation of the solid skeleton requires the application of a large strain theory. The irreversible time-dependent behaviour is described by an elastic-viscoplastic constitutive model. Usually the deformation gradient is multiplicatively split into an elastic and an inelastic part. Thereby, an intermediate configuration arises. The assumption of small elastic strains enables the establishment of the constitutive equations on the current configuration. A hyperelastic strain energy function of Neo-Hookean type is applied to capture the reversible part of the deformation. In addition, irreversible strains are produced under compressive loads. The anisotropic viscoplastic model is formulated in principal stresses. The rate of inelastic strains depends on the state of stresses as well as on the density expressed by internal variables. The matric suction, the difference between gas and liquid pressure, is included by the van GENUCHTEN approach. The intrinsic permeability tensor of the solid skeleton is anisotropic and alters during the deformation process.

BUBNOV-GALERKIN Finite Element Method is combined with a Finite Volume Method based approach in order to solve the coupled initial-boundary value problem simultaneously. Thus, equation (2) is pulled back to the reference configuration and transformed into its weak form by the *Principle of Virtual Work*. Furthermore, liquid pressure and gas saturation are chosen as primary variables for the fluid transport. The mass balances, equations (1), are converted applying the box-method, Helmig [4]. A fully up-winding scheme stabilises the advective dominated transport equations. The discrete problem is solved by a NEWTON-RAPHSON procedure. Selected examples demonstrate the capacity of the chosen approach.

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