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Blended Hermite MLS approximation for discretizing biharmonic partial differential equations

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ABSTRACT

Meshless methods are an appealing choice for constructing functional approximations (with different degrees of consistency and continuity) without a mesh support. Some members of the vast family of meshless methods are the smooth particle hydrodynamics (SPH) proposed in , the element free Galekin (EFG) , the diffuse elements (DE) , the reproducing kernel particle method (RKPM) , the natural element method and others.

In this work we are going to perform a deeper analysis of Hermite moving least square based approximations. Here, we are analyzing its use in the solution of fourth order partial differential equations usually encountered in structural mechanics involving beams or plates.

It is well known that in the framework of the more experienced discretization techniques the treatment of fourth order PDE induces some issues. One possibility to treat these equations lies in the use of collocation techniques. In simple geometrical domains finite differences schemes could be applied as well as other schemes based on use of pseudo-spectral collocation techniques. Of course these techniques only implies one degree of freedom per node, but the enforcement of boundary conditions deserve some further developments because two conditions must be simultaneously enforced at the nodes on the domain boundary. The most common approach consist of enforcing the second boundary condition by penalization or by introducing a Lagrange multiplier. However, the use of collocation techniques was restricted to simple geometrical domains. In the more general case variational formulation were widely adopted.

We would like to emphasize that within the meshless approximation framework one could derive fourth order derivatives formulas (because the excellent continuity properties of usual meshless approximations) to be used in the discretization of fourth order PDE in complex geometries. However, this potentiality has not been sufficiently explored, in our knowledge, to conclude about its benefits and weakness. In any case, the issue related to the extra-boundary conditions persists.

Within the variational framework it is well known that first order continuity of the field approximation is needed to discretize the weighted residual formulation after integration by parts twice. In the finite

element context this fact implies the use of Hermite approximation whose more peculiar consequence is the necessity to introduce also the field derivatives as nodal degrees of freedom. Thus, the continuity of the field derivatives across the element boundaries is ensured preserving the fully consistency of the discrete form. Moreover in this context the imposition of the two boundary conditions results natural. The prize to pay is the increase of the number of degrees of freedom involved in the discret model.

With the advent of meshless techniques the just referred issue was revisited. Most part of meshless approximations has an adjustable degree of continuity. For moving least squares based techniques (RKPM, EFG or ED) this is quite obvious. Thus, one could expect that the use of any of these approximations should solve the aforementioned continuity issues.

Thus two possibilities come: (i) using a standard meshless approximations everywhere in the whole domain and then enforce the boundary condition by invoking an appropriate technique (penalization, Lagrange multipliers, Nitsche, ...) or (ii) using an Hermite MLS based approximation everywhere in the whole domain and enforce the boundary conditions by direct collocation at nodes located on the boundary (as in the finite element framework). Obviously the second alternative seems "a priori" more natural to address the boundary conditions issue whereas the first one seems more appealing from the computational point of view because it limits the number of degree of freedom at each node.

We present a mixed approach that consists of a standard MLS approximation inside the domain but that becomes a Hermite-MLS when the domain boundary is approached. Thus, only extra-degrees of freedom are introduced at the nodes located in the boundary neighborhood. Thus we could: (i) reduce drastically the number of degrees of freedom of the discreet model and then the CPU time; (ii) compare a simple enforcement of boundary conditions at nodes located on the boundary with the enforcement of such boundary conditions by using a penalty strategy (the use of Lagrange multipliers and the Nistche technique is a work in progress) and (iii) compare the error and convergence rates of a fully Hermite approximation, a standard MLS one and the mixed formulation here proposed, when boundary conditions are treated exactly in the same manner (using two penalty coefficients) and also compare these results with the ones coming from a direct nodal enforcement of the boundary conditions.

The just referred discussion will be carried out by considering both 1D and 2D models and a rich enough Gauss numerical integration.