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IMMERSED PRESSURE LOSS BOUNDARY CONDITION ON CARTESIAN GRID

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ABSTRACT

Recently, a Cartesian grid method is widely used for the computation of the flow around a complex geometry. On a Cartesian grid, the size of the grid width is of great significance to solve flows, enough grid points, however, are not available due to the restriction of computer resources and the limitation of the calculation time in most computation. For example, in the case of the flow around porous media that have very fine structure relevant to the smallest grid size, it is important to model the macroscopic fluid property of the porous media. Conventionally, these properties are modeled by analytical or empirical formulas and are build onto a governing equation like the Darcy's law.

Heat exchangers are a common fluid component and are equipped in the fluid machinery or the automobile that have very complex shape. The heat exchanger acts as a porous medium against the fluid and has a well-established fluid property, i.e. the relationship between the pressure loss and the mass flow rate passing through the heat exchanger. To treat the heat exchanger that is unaligned to the underlying Cartesian grid, it is required to control the gradient direction of the pressure loss as well as the direction of outflow from the heat exchanger. In this paper, the implementation of the pressure loss effect onto the Navier-Stokes equations is discussed based on the study of the irregular boundary treatment for Poisson equation on Cartesian grid [1].

The governing equations are the incompressible NS equations and the continuity equation. The NS equations are described by Euler explicit scheme as follows.

$$\frac{\partial u_i}{\partial t}^{n+1} + \frac{\partial}{\partial x_j}(u_i u_j)^n = -\frac{\partial p}{\partial x_i}^{n+1} + \frac{1}{\mathrm{Re}}\frac{\partial}{\partial x_j}\left(\frac{\partial u_i^n}{\partial x_j}\right) \tag{1}$$

Note that the effect of the pressure loss inside the heat exchanger is described by S_i, which includes all momentum loss at the heat exchanger, and then the equations inside the heat exchanger are given by:

$$\frac{\partial u_i}{\partial t}^{n+1} + \frac{\partial}{\partial x_j}(u_i u_j)^n = -\frac{\partial p}{\partial x_i}^{n+1} + S_i^{n+1} \tag{2}$$

Introducing Heaviside function $H(x_i)$ to distinguish between the heat exchanger and the fluid cell, the momentum equations can be rewritten as:

$$\frac{\partial u_i^{n+1}}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j)^n = -\frac{\partial p^{n+1}}{\partial x_i} + \frac{1-H(x_i)}{\mathrm{Re}}\frac{\partial}{\partial x_j}\left(\frac{\partial u_i^n}{\partial x_j}\right) + H(x_i)\, S_i^{n+1} \tag{3}$$

where $H(x_i)$ =0 and $H(x_i)$ =1 indicate the fluid and the heat exchanger, respectively. The pressure loss at the heat exchanger is modeled as follows.

$$S_i^{n+1} = -\,sgn(u_i^{n+1})\frac{\Delta p_{hex}^{n+1}}{\Delta r} n_i\,, \quad \Delta p_{hex}^{n+1} = f(u_i^{n+1}) \tag{4}$$

where Δp, Δr, and n_i represent the pressure loss, the thickness and the normal vector of the heat exchanger, respectively. The pressure loss is defined by the function of the velocity passing through the heat exchanger, of which coefficients are obtained from an experiment. The equation is solved by the fractional step method with a relaxation process both pressure and velocity fields all together like HSMAC method. Equations (1)~(4) indicate that the pressure loss is implicitly treated in the relaxation process.

Verification was performed on several cases. First example is the flow around inclined heat exchangers with three different angles. Fig. 1 shows the flow fields and table 1 indicates the errors. It was found that the proposed technique could reproduce appropriate flow fields. Second example as depicted in Fig. 2 shows the comparison for the heat exchangers with three different tilt angles that keep the same relative angle with the flow direction. Computed results in table 2 indicate that all calculation results are almost same and within 3 percent error against Eq.(4), i.e., $\Delta p=0.8u^2_{hex}$.

A technique to treat the pressure loss boundary condition correspond to an unaligned shape against the Cartesian grid is proposed through the external force in the incompressible NS equations. It was confirmed that this method was powerful to predict the flow that contains the pressure loss components like a heat exchanger.

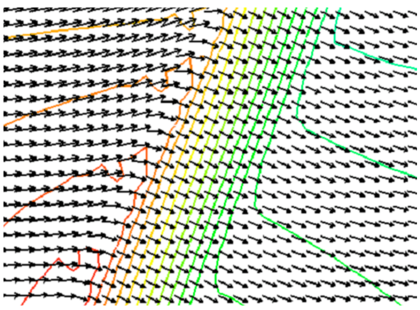

Fig. 1 Velocity vectors and pressure contour in case of 25 degrees inclined.

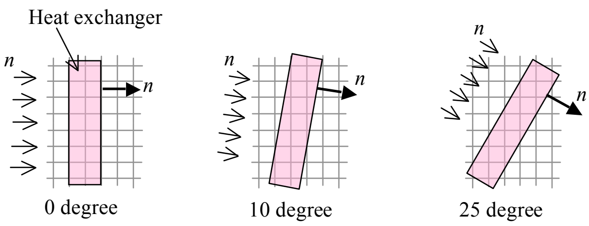

Fig. 2 Schematic of computed heat exchangers with three different tilt angles.

Table 1 Verification results of velocity and pressure loss for different tilt angles.

Angle	10 deg.	25 deg.	45 deg.
u_{hex}	0.793	0.710	0.587
Δp	0.492	0.395	0.284
Δp Eq.(4)	0.503	0.403	0.276
Error [%]	2.3	2.1	2.9

Table 2 Comparison results for constant relative position between inflow and heat exchanger.

Angle	0 deg.	25 deg.	45 deg.
u_{hex}	0.762	0.769	0.771
Δp	0.460	0.463	0.465
Δp Eq.(4)	0.465	0.473	0.476
Error [%]	1.0	2.2	2.3

REFERENCES

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