

## NUMERICAL SIMULATION AND SENSITIVITY ANALYSIS OF THERMALLY INDUCED FLOW INSTABILITIES

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### ABSTRACT

We present computation of transient flow and heat transfer of generalized Newtonian fluids. Flow sensitivities with respect to problem parameters are computed using the continuous sensitivity equation method [1]. Simple flow in symmetric gate geometries (Fig. 1) exhibits symmetric and non-symmetric solutions depending on the values taken by flow parameters. Generally the flow is stable. However, for certain combinations of operating conditions and gate geometry causing large thermally induced viscosity gradients, symmetric flows occur. This phenomenon is very important for injection molding of plastic or metal powder materials because an unstable flow leads to defective parts [2]. We discuss the ability of the algorithm to predict the transition boundary between stable and unstable flows and the behavior of the solution sensitivity in critical transition regions.

Cooling such a fluid can cause thermally induced flow instability since the viscosity depends on temperature and shear rate. We use a power law model  $\eta = C\dot{\gamma}^m e^{-bT}$  where  $C$ ,  $m$  and  $b$  are model constants. A small decrease in temperature on one side relative to the other will cause an increase in viscosity that will in turn decrease flow and allow further localized cooling. Under certain circumstances, these changes will cascade until the observed flow ceases in one branch and accelerates in the other. Fig. 2 shows the velocity distribution for an unstable flow: the flow enters by the bottom section and should normally exit with the same flow rate by the left and right outlets. Here the flow has stopped on the left exit and accelerated on the right.

Dimensional analysis shows that for a given power law coefficient  $m$ , the flow and heat transfer are determined by two dimensionless quantities, namely the Graetz number  $Gz = (\rho c_p U_0 H^2)/(kR)$  (ratio of heat conduction time and filling time) and  $B = b\Delta T_0$  (indicating the sensitivity of viscosity to temperature change). Numerical results were obtained for  $m = -0.8$  and various values of  $Gz$  and  $B$ . Numerical solutions indicate that the flow becomes non-symmetric for specific combinations of  $Gz$  and  $B$  numbers as shown in Fig. 3. Notably the flow is unstable for moderate values of the  $Gz$  number and large values of  $B$ . Remark also a region of stable flow for very small values of  $Gz$ .

Flow sensitivities indicate how the solution responds to changes in flow parameters. Fig. 4 shows the temperature difference between two points symmetrically located in the mid-section of the left and right outlets (red curve) and its sensitivities. For this case the flow is unstable: the temperature is higher on the right hand side and the flow accelerates on this side further increasing the temperature difference. A positive sensitivity indicates that the temperature difference increases when increasing the value of the parameter, whereas a negative sensitivity indicates the opposite effect. Fig. 4 shows that the temperature difference decreases when increasing  $m$  (viscosity less dependent on the shear rate) and increases for higher  $B$  (larger temperature dependence of the viscosity). When increasing the  $Gz$  number, the temperature difference will first become smaller and then bigger than for the actual value of  $Gz$ .

Flow and sensitivity analysis can be used to provide guidelines for design and operation of diaphragm gates in order to avoid combinations of parameters that fall in the unstable flow region. To move from an unstable to a stable flow, one may change operating conditions: (1) increase injection speed, (2) increase mold temperature or (3) decrease molding temperature; to make mold design changes as: (1) increase gate thickness, (2) decrease gate flow path; or material changes by selecting a material with (1) lower thermal diffusivity, (2) lower temperature dependence of the viscosity and (3) less shear thinning.

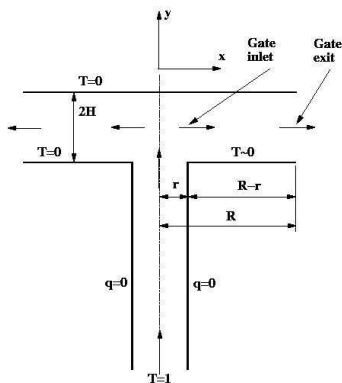


Fig. 1: Model problem

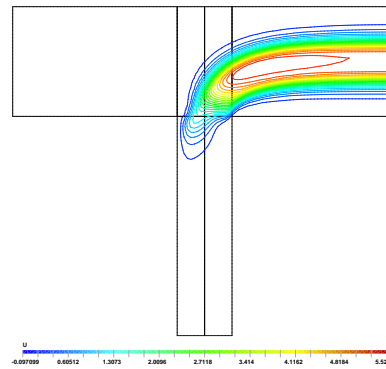


Fig. 2: Velocity distribution for unstable flow

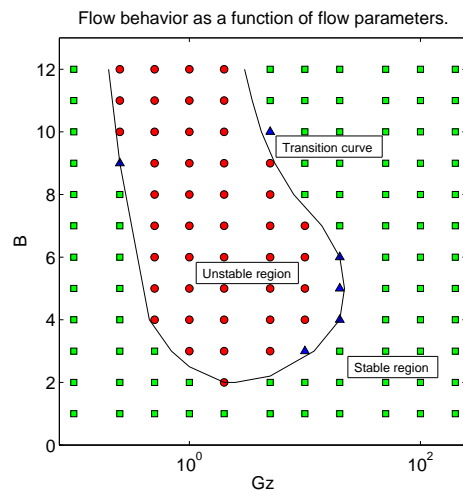


Fig. 3: Stability diagram in parameter space

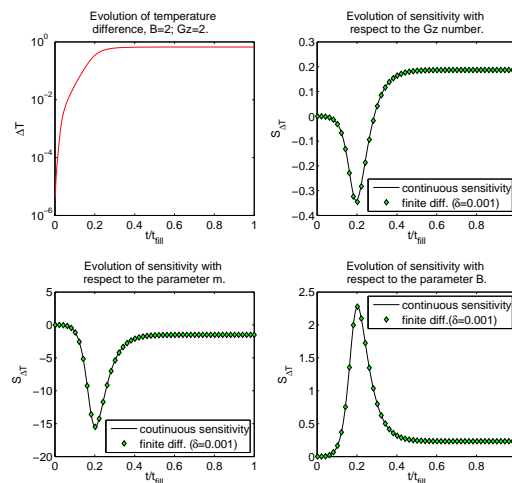


Fig. 4: Temperature difference and its sensitivities

## REFERENCES

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