

ADAPTIVITY, SENSITIVITY AND UNCERTAINTY IN VERIFICATION AND VALIDATION

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ABSTRACT

This paper shows that adaptive methods are a cost-effective and simple tool to achieve grid independent or verified solutions. The Zhu-Zienkiewicz error estimates come at negligible cost so that they can be viewed as single grid error estimators. This is in contrast with the Grid Convergence Index which requires a minimum of 2 grid solutions. It also explores the Sensitivity Equation Method (SEM) as a tool to perform sensitivity and uncertainty analysis of CFD predictions. These approaches constitute powerful tools to perform Verification and Validation of CFD predictions and build confidence in CFD predictions. The resulting uncertainty bars put CFD *on par* with experimental techniques. Adaptivity is discussed using the $k - \epsilon$ model of turbulent flows as an example. The SEM is applied to the $k - \epsilon$ model, to free convection with variable fluid properties, and to some fluid-structure interaction problems. Uncertainty intervals are predicted and compared to measurements. Taken together, these approaches offer good prospect for developing families of computing methods that can be viewed as *standards* of good practices in CFD.

The present paper illustrates how numerical errors can be controlled via mesh adaptation to provide solutions of high quality and accuracy. It also shows how Sensitivity Analysis can yield estimates of uncertainty of the flow response to uncertainties or inaccuracies in the data input to the flow solver. The former provides a tool for assessing to what extent the differential equation has been solved "exactly". The latter provides a tool to assess to what extent these *accurate* solutions can be trusted. The larger the uncertainty of the flow response the lower the confidence.

The present paper should be viewed in the light of efforts on verification and validation as discussed in Roache's book [1]. In this book, Verification is defined as a synonym for solving the equations accurately (*Solving the equations right*). Hence it is a mathematical and numerical analysis exercise. It proceeds in two steps. First, the methods of manufactured solutions [1] is used for code verification. In the present context, the automated adaptive grid refinement allows the simultaneous verification of the flow and sensitivity solvers, the error estimation module, and the adaptive remeshing strategy. Secondly, the verified code is applied to the practical problem of interest. Mesh adaptation provides a simple way of performing the grid refinement studies required to verify the simulation

Validation is defined a process to determine if the right equations are solved for the process at hand (*Solving the right equations*). It is essentially an engineering activity involving competition with laboratory or field data. Here, we provide one additional step: sensitivity analysis is used to provide uncertainty intervals for the CFD solution computed at the nominal values of the parameters.

Error estimation and grid adaptation are those described in [2] for both laminar and turbulent flows variables and their sensitivities. The techniques for obtaining flow sensitivities are described in [2] and [3]. Uncertainty intervals are obtained by using the flow sensitivities to cascade input data uncertainties through the CFD code to yield uncertainty estimates of the flow response.

The final version of the paper will contain details about the ZZ error estimator and its variant that provide assessment of the error on integral quantities such as lift and drag coefficients.

Uncertainty analysis of CFD results provides a rigorous framework for comparing predictions to measurements (validation of predictions). The resulting uncertainty bars put CFD *on par* with experimental techniques. Thus, comparison of predictions with measurements becomes a more rigorous and meaningful exercise.

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