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ACCURATE UPPER BOUND OF THE DISCRETIZATION ERROR IN XFEM THROUGH A RECOVERY-BASED TECHNIQUE AND ERROR ESTIMATION OF THE RECOVERED SOLUTION

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ABSTRACT

In the FEM, the ability of the *a posteriori* implicit residual-based error estimators to produce guaranteed error bounds has traditionally been an advantage over the recovery-based estimators [1],[2], which, on the other hand, are usually preferred by practitioners due to their simple implementation and robustness. However, Díez, Ródenas and Zienkiewicz [3] have recently developed a recovery-type error estimator based on an equilibrated patch recovery technique that also provides accurate upper bounds of the error.

The extended finite element method (XFEM) has emerged as a highly efficient numerical technique for the analysis of linear elastic fracture mechanics (LEFM) problems. Like in the FEM, the XFEM results are also affected by the discretization error. Hence, error assessment and bounding tools are also required for XFEM. Ródenas *et al.*[4] proposed an adaptation to the XFEM framework of the recovery-based procedure introduced by Díez *et al.*[3] for the FEM framework.

This paper presents both, an improvement of the technique described in [4] which provides sharp upper bounds of the error in XFEM and an error estimator for the recovered stress field $\boldsymbol{\sigma}^*$. The upper bound property requires $\boldsymbol{\sigma}^*$ to be statically equilibrated and continuous. As described in [4], the equilibrium restrictions are exactly satisfied at patches by using a special recovery technique whereas continuity is enforced using a partition of unity postprocessing which introduces small lacks of equilibrium. We propose the following expression to compute and account for these equilibrium defects in the evaluation of the error upper bound of the exact error $\|\mathbf{e}\|$.

$$\|\mathbf{e}\|^2 \leq \int_\Omega \left(\boldsymbol{\sigma}^* - \boldsymbol{\sigma}^h\right)^T \mathbf{D}\left(\boldsymbol{\sigma}^* - \boldsymbol{\sigma}^h\right) d\Omega - 2\int_\Omega \mathbf{e}\cdot\mathbf{s}\, d\Omega - 2\int_\Gamma \mathbf{e}\cdot\mathbf{r}\, d\Gamma = E_{UB} \tag{1}$$

Where the first integral is the estimated error in energy norm provided by $\boldsymbol{\sigma}^*$, the last two integrals represent the corrections that account for the lacks of internal equilibrium $\mathbf{s}$ (see [3]) and contour equilibrium $\mathbf{r}$ corresponding to $\boldsymbol{\sigma}^*$ and $\mathbf{e}$ is the error of the displacements field, for which an approximation has been obtained as described in [4].

The classical Westergaard problem in mode I analyzed with a sequence of uniformly refined meshes has been used to verify the behaviour of (1). Figure 1.a) represents the effectivity index corresponding to the first integral in (1), curve $SPRC_{XFEM}$ and the effectivity index of the upper bound $E_{UB,}$ curve *UB*. The results show that (1) can be used to obtain sharp computable upper bounds of the error in energy norm. It can also be shown that the last two integrals in (1) can be used to estimate the error in energy norm associated to $\boldsymbol{\sigma}^*$. Figure 1.b) shows the evolution of different errors in energy norm ($\|\mathbf{e}_{ex}\|$ corresponds to the exact error, $\|\mathbf{e}_{es}\|$ is the error estimated by the 1st integral in (1), $\|\mathbf{e}^*_{ex}\|$ is the exact error associated to $\boldsymbol{\sigma}^*$ and $\|\mathbf{e}^*_{es}\|$ is the estimation of this error provided by the last two integrals in (1)). The graph shows that $\|\mathbf{e}^*_{es}\|$ captures the order of magnitude of $\|\mathbf{e}^*_{ex}\|$ and its slope. The results obtained with other problems have shown the same pattern. This shows that this is a promising technique for estimating $\|\mathbf{e}^*_{ex}\|$, although further tests are required on this subject.

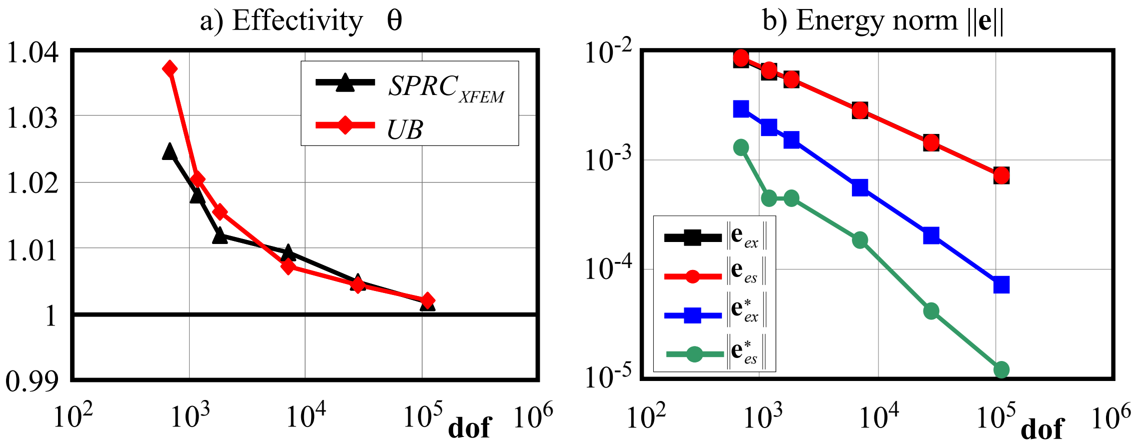

Figure 1.- Effectivity results and error convergence curves obtained with equation (1)

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