8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

ENERGY METHODS FOR BOUNDARY CONDITIONS IDENTIFICATION FOR DYNAMIC PROBLEMS

***Thouraya N. Baranger[1], Stéphane Andrieux[2]**

[1] LaMCoS, INSA-Lyon,
CNRS UMR5259, F69621, France ;
Université de Lyon, Lyon, F-69003, France;
Université Lyon 1,
Villeurbanne, F-69622, France
Thouraya.Baranger@univ-lyon1.fr

[2] LaMSID, UMR CNRS-EDF 2832,
1, avenue du Général de Gaulle, 92141
Clamart, France
stephane.andrieux@edf.fr

Key Words: *Identification, inverse Problems, Dynamic, Damping, Cauchy problem, Boundary Conditions.*

ABSTRACT

The recent and rapid development of full surface field imaging gives a new topicality to the classical problem, known as the Cauchy problem, of expanding a field, known together with it dual quantity field on a part a boundary of a solid, towards the interior of it. In the applications, the problem is seen as a data completion problem that is a search for lacking information on a part of the boundary of a solid provided overspecified data are available on another part of the boundary. Various applications can then be identified such as inverse problem applications (identification of unknown boundary conditions, geometrical inverse problems, identification of material parameters in a geometrically know inclusion embedded in an elastic solid, estimation of contact zones…) or damage detection or distributed material parameters identification (the data completion being the first step of the identification procedure). But direct imaging applications could also be envisaged complementing 3D tomography imaging techniques, which give on geometrical information, with quantitative estimation of stress fields.

A two-fields based energy error method has been designed for general symmetric elliptic operators, leading to efficient numerical algorithms that have been implemented in various situations, including 3D heterogeneous ones with non linear boundary conditions (Andrieux and Baranger (2006, 2007)). The method is based on the definition of an appropriate energy error functional, function of the fields on the boundary that are out of reach. This functional turns out to be positive quadratic and its (null) minimum delivers the desired data when the available data on the boundary where the Cauchy problem is stated are compatible. Furthermore, the computation of the functional and its gradient is greatly simplified by an alternative expression involving only the boundary of the solid. Turning now towards hyperbolic equations on a time domain $[0, T]$ for structural damped dynamics problems, the question of extending the energy error functional is investigated. In this context, the Cauchy problem is defined as follows, find $\boldsymbol{u}(x,t)$ such that :

$$
\begin{cases}
\rho\ddot{\boldsymbol{u}} + c\dot{\boldsymbol{u}} - div(\mathbf{A}:\varepsilon(\boldsymbol{u}) + \mathbf{B}:\varepsilon(\dot{\boldsymbol{u}})) = 0 & & in\ \ \Omega\times]0,T[\\
[\mathbf{A}:\varepsilon(\boldsymbol{u}) + \mathbf{B}:\varepsilon(\dot{\boldsymbol{u}})].\boldsymbol{n} = \boldsymbol{T}^m, & \boldsymbol{u} = \boldsymbol{U}^m & on\ \ \Gamma_m\times]0,T[\\
[\mathbf{A}:\varepsilon(\boldsymbol{u}) + \mathbf{B}:\varepsilon(\dot{\boldsymbol{u}})].\boldsymbol{n} = \boldsymbol{T}^u, & \boldsymbol{u} = \boldsymbol{U}^u & on\ \ \Gamma_u\times]0,T[\\
\boldsymbol{u}(x,0) = \bar{\boldsymbol{u}}^{0}(x,0), & \dot{\boldsymbol{u}}(x,0) = \bar{\boldsymbol{u}}^{1}(x,0) & in\ \ \Omega
\end{cases}
\tag{1}
$$

Here, Γ_m is the boundary where the overspecified data are measured and Γ_u is the boundary where the data ($\boldsymbol{T}^u$, $\boldsymbol{U}^u$) have to be identified, c is the viscous damping and **B** is more general viscous bulk tensor. Then, the error functional in this dynamic setting acts on the difference of two fields u_1 and u_2 and is composed of two terms. The first one is the reversible and kinetic energy at the final time instant; the second one is the integral over the space-time domain of the dissipation power:

$$
J(w) = \frac{1}{2}\int_{\Omega} \rho\dot{w}^2 + A:\varepsilon(w):\varepsilon(w)\Big|_{t=T} + \frac{1}{2}\int_0^T\int_{\Omega}[c\dot{w}^2 + B:\varepsilon(\dot{w}):\varepsilon(\dot{w})]
$$

with: $w = u_1 - u_2,\ \dot{w} = \dot{u}_1 - \dot{u}_2$ (2)

$$
\begin{cases}
\rho\ddot{\boldsymbol{u}}_i + c\dot{\boldsymbol{u}}_i - div(\mathbf{A}:\varepsilon(\boldsymbol{u}_i) + \mathbf{B}:\varepsilon(\dot{\boldsymbol{u}}_i)) = 0 & \text{in}\ \ \Omega\times]0,T[& \text{for } i = 1,2 \\
[\mathbf{A}:\varepsilon(\boldsymbol{u}_i) + \mathbf{B}:\varepsilon(\dot{\boldsymbol{u}}_i)].\boldsymbol{n} = \boldsymbol{T}^m, \text{ if } i = 2 \text{ and } & \boldsymbol{u}_i = \boldsymbol{U}^m \text{ if } i = 1 & \text{on } \Gamma_m\times]0,T[\\
[\mathbf{A}:\varepsilon(\boldsymbol{u}_i) + \mathbf{B}:\varepsilon(\dot{\boldsymbol{u}}_i)].\boldsymbol{n} = \boldsymbol{T}^u, \text{ if } i = 1 \text{ and } & \boldsymbol{u}_i = \boldsymbol{U}^u \text{ if } i = 1 & \text{on } \Gamma_u\times]0,T[\\
\boldsymbol{u}_i(x,0) = \bar{\boldsymbol{u}}^{0}(x,0),\ \dot{\boldsymbol{u}}_i(x,0) = \bar{\boldsymbol{u}}^{1}(x,0) & \text{in}\ \ \Omega & \text{for } i = 1,2
\end{cases}
$$

The B.C identification problem is then formulated as follows:

$$
\underset{T^u,U^u}{Min}\ \ J(u_1(T^u) - u_2(U^u)) \tag{3}
$$

However, for undamped problem ($\boldsymbol{B} = c = 0$), the exact controllability result imposes to supplement the error functional with an extra penalization term on the boundary to recover the property of identifiability of the lacking boundary conditions:

$$
J_{\varepsilon}(w) = \frac{1}{2}\int_{\Omega} \rho\dot{w}^2 + A:\varepsilon(w):\varepsilon(w)\Big|_{t=T} + \frac{\varepsilon}{2k}\int_0^T\int_{\partial\Omega} w^2 \tag{3}
$$

These new results will be discussed and illustrated on various applications and compared to the algorithm developed by Kozlov *et al* (1991) and Baumeister *et al*.(2001).

REFERENCES

[1] S. Andrieux and T. N. Baranger, An energy error based method for the resolution of the Cauchy problem in 3D linear elasticity, *Computer Methods in Applied Mechanics and Engineering*, DOI: 10.1016/ j.cma. 2007.08.022 . 2007.

[2] T. N. Baranger and S. Andrieux., An Optimization Approach to solve Cauchy Problem in linear elasticity, *Journal of Structural and Multidisciplinary Optimization*, DOI 10.1007/s00158-007-0123-5, 2007.

[3] S. Andrieux, T.N. Baranger, A. Ben Abda, Solving Cauchy problem by minimizing an energy-like functional, *Inverse Problems*, 22, (2006), 115-133.

[4] J. Baumeister and A Leitão,, On iterative methods for solving ill-posed problems modelled by partial differential equations, *J. of Inverse and Ill-posed problems*, 9, pp13-30, 2001

[5] V.A Koslov., V.G Maz'ya. and A.V Fomin. An iterative method for solving the Cauchy problem for elliptic equations, *Comput. Meth. Math. Phys.*, Vol. 31, N°1, (1991), 45-52