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TWO GEOMETRIC INTEGRATORS FOR FINITE-STRAIN VISCOPLASTICITY WITH KINEMATIC HARDENING

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ABSTRACT

We analyze the numerical treatment of a viscoplastic material model of overstress type. The material model is based on a double multiplicative split (see [4], [6]) and takes the nonlinear kinematic hardening into account. According to the kinematic assumptions of the double multiplicative split, symmetric tensor-valued internal variables $\mathbf{C}_{\mathrm{i}}, \mathbf{C}_{\mathrm{ii}} \in Sym$ describe the current state of the material. For a given deformation history $\{\mathbf{C}(t)\}_{t\in[0,T]}$, the following thermodynamically consistent evolution equations govern the dissipative processes at the material point:

$$\dot{\mathbf{C}}_{\mathrm{i}} = \mathbf{f}_{\mathrm{i}}(\mathbf{C}(t), \mathbf{C}_{\mathrm{i}}, \mathbf{C}_{\mathrm{ii}})\ \mathbf{C}_{\mathrm{i}}, \quad \dot{\mathbf{C}}_{\mathrm{ii}} = \mathbf{f}_{\mathrm{ii}}(\mathbf{C}(t), \mathbf{C}_{\mathrm{i}}, \mathbf{C}_{\mathrm{ii}})\ \mathbf{C}_{\mathrm{ii}}. \tag{1}$$

Under appropriate initial conditions, the exact solution of (1) has the following geometric property: $\mathbf{C}_{\mathrm{i}}$ and $\mathbf{C}_{\mathrm{ii}}$ lie on the manifold M, defined by

$$M := \big\{\mathbf{B} \in Sym : \mathrm{det}\mathbf{B} = 1\big\}. \tag{2}$$

In particular, this implies that the inelastic flow is incompressible: $\mathrm{det}\mathbf{C}_{\mathrm{i}} = 1$, which is a typical feature of metal plasticity. We note that some strong local nonlinearities are peculiar to the problem (1). Particulary, the forcing function $\mathbf{C}(t)$ is continuous, but not smooth. Another source of the strong nonlinearity is due to the distinction into elastic and inelastic material behaviour.

We discuss the implementation of the material model to the displacement-based finite element method (FEM). A global implicit time stepping procedure in the context of FEM requires a proper stress algorithm, which is based on the implicit integration of (1). We show that the Modified Euler-Backward Method (MEBM) (see [3], [6]) and the implicit Exponential Method (EM) (see [5], [1]) can be used to construct a geometric integrator of (1), such that the numerical solution remains on $M \times M$. The excellent accuracy and convergence characteristics of MEBM and EM are demonstrated via special numerical tests.

It is axiomatic that if the numerical solution leaves the manifold $M \times M$, it will introduce non-physical degrees of freedom, and some structural features of the flow will be lost. In this connection, we assess those factors that result in a more accurate computations compared to the classical Euler-Backward method, especially when integrating with big time steps and for long times:

- The numerical error of the classical Euler-Backward method, related to the violation of incompressibility condition, tends to accumulate over time (see, for example, [1], [3]).
- If the incompressibility constraint is violated, the straight-forward computation of stresses will result in a wrong hydrostatic stress. In that case, corrections of the hydrostatic stress are required.
- Using a series of numerical tests of MEBM and EM, we show that the the numerical error is not accumulated over time.
- The strong local nonlinearities may lead to essential integration errors. Once committed, these errors are reduced with time if the geometric integrators are implemented.

REFERENCES

[1] W. Dettmer, S. Reese. “On the theoretical and numerical modelling of Armstrong-Frederick kinematic hardening in the finite strain regime”. *Comp. Meth. in Appl. Mech. and Eng.*, Vol. **193**, 87–116, 2004.

[2] E. Hairer, C. Lubich and G. Wanner. *Geometric Numerical Integration. Structure Preserving Algorithms for Ordinary Differential Equations.*, 2nd. Edition, Springer, 2004.

[3] D. Helm. “Stress computation in finite thermoviscoplasticity”. *Int. J. of Plast.*, Vol. **22**, 1699–1721, 2006.

[4] A. Lion. “Constitutive modelling in finite thermoviscoplasticity: a physical approach based on nonlinear rheological elements”. *Int. J. of Plast.*, Vol. **16**, 469–494, 2000.

[5] C. Miehe, E. Stein. “A canonical model of multiplicative elasto-plasticity: formulation and aspects of the numerical implementation”. *Europ. J. of Mech. A/Solids.*, Vol. **11**, 25–43, 1992.

[6] A. V. Shutov, R. Kreißig “Finite strain viscoplasticity with nonlinear kinematic hardening: Phenomenological modeling and time integration”. *Comp. Meth. in Appl. Mech. and Eng.*, accepted for publication, 2007.