

## DUALITY IN THE GEOMETRICALLY EXACT ANALYSIS OF REISSNER-SIMO BEAMS

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### ABSTRACT

An hybrid-mixed variational formulation for the geometrically exact analysis of elastic rods, modelled by the Reissner-Simo theory, is presented. This formulation is based on an extension of the Principle of Complementary Energy, which has a symmetric tangent bilinear form (tangent operator), leading to a symmetric tangent matrix of the corresponding finite element model.

Furthermore the adopted finite element discretization produces numerical solutions that strongly satisfy the equilibrium differential equations in the elements, as well as on the static (Neumann) boundary conditions, being therefore a true equilibrium formulation for arbitrarily large displacements.

As this formulation does not suffer shear locking or any other artificial stiffening phenomena, they may be regarded as an alternative to conventional displacement elements, based on the stationarity of the potential energy functional.

Nevertheless the main advantage of this kind of solution results from their application in the context of error estimation, pairing their results with those obtained from displacement formulations.

Examples of the analysis of planar and three-dimensional structures will be presented, including the analysis of their response and studies of the convergence.

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