8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Non-Deterministic Compressible Navier-Stokes Simulations Using Polynomial Chaos

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Key Words: *Polynomial Chaos, CFD, Navier-Stokes, Compressible*

ABSTRACT

With the increasing use of CFD as a design tool, and the associated requirements of accuracy and reliability, came also the need for assessing the effect of uncertainties in the CFD model. These uncertainties can be of various nature: uncertainty on boundary conditions, on physical variables (such as e.g. viscosity or conduction), on geometry (e.g. manufacturing tolerance), on CFD modeling variables (e.g. constants of turbulence models) etc.

Several methods exist to deal with these uncertainties. The present paper will focus on the Polynomial Chaos (PC) method.

This methodology was first formulated by Wiener, [1]. Xiu and Karniadakis, [2], extended the PC method to the so-called generalized PC where a variety of basis functions can be used, according to the Askey scheme.

Whereas the use of the PC method is well established in structural mechanics its application to CFD problems is more recent, e.g. [3], **Error! Reference source not found.**, [4], [5].

The classical PC method is an intrusive methodology in the sense that the governing equations are altered. In the present contribution this intrusive method is applied to the compressible Navier-Stokes equations.

Several approaches to make the method more efficient will be discussed, such as the use of a pseudo-spectral approach and a non-time accurate formulation for steady state applications.

To illustrate the approach results for a lid driven cavity flow at Reynolds number Re=100 are shown. The 2D laminar Navier-Stokes equations were solved with the PC method on a mesh of 40x40 uniform cells. The viscosity is considered as uncertain variable with a Gaussian distribution and a standard deviation of 10%. A 2nd order pseudo-spectral PC approach is used.

The PC results give the full statistical information on the flow, with the local PDF (probability density function) of all flow variables. This is illustrated in Figure 1

showing the calculated mean velocity components (u_0 and v_0) along the centerlines x=0.5 and y=0.5. On the plot the associated uncertainty is indicated with an error bar corresponding to the 90% interval resulting from the local PDF. Note that the error bar is not necessarily symmetric with respect to the mean value; this corresponds to a non-symmetric PDF.

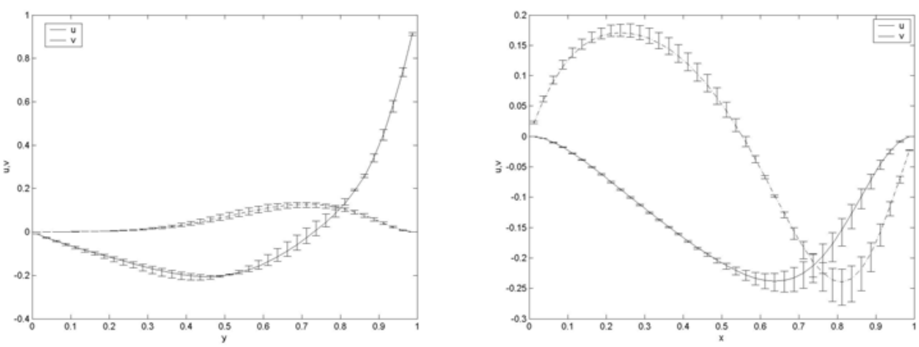

Figure 1 Mean values of u, v along the centrelines and the associated uncertainty indicated with error bars: left x=0.5;right y=0.5

Currently the extension to turbulent Navier-Stokes has been implemented and PC simulations for the NASA Rotor 37 are underway. These results will be included in the full paper.

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