8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Stochastic Modal Analysis of Structures with Random Shape Using X-FEM

*** Severine Lepage[1], Laurent Van Miegroet[2] and Pierre Duysinx[3]**

[1] Beckman Institute
University of Illinois
at Urbana-Champaign
405 N. Mathews Av.,
Urbana, IL. 61801, USA
slepage@uiuc.edu

[2] Aerospace and Mechanical
Engineering Department
University of Liege
Chemin des chevreuils, 1,
B-4000 Liege, Belgium
L.VanMiegroet@ulg.ac.be

[3] Aerospace and Mechanical
Engineering Department
University of Liege
Chemin des chevreuils, 1,
B-4000 Liege, Belgium
P.Duysinx@ulg.ac.be

Key Words: *Stochastic Finite Element Method, eXtended Finite Element Method, Stochastic Eigenvalue Analysis, Level Set description.*

ABSTRACT

Manufacturing processes may leave substantial variability in the shape and geometry of devices, which leads to uncertainty in their performance and reliability. Therefore, in order to efficiently design high performance structures, the effects of these geometric variations have to be considered and a modeling methodology is needed to ensure required device performance under uncertainties. The purpose of this work is to develop a numerical method to analyze the effects of random shape variations on the structural natural frequencies using a probabilistic eXtended Finite Element Method (X-FEM [1]).

The present work focuses on second moment approaches, in which the first two statistical moments, i.e. the mean and the variance, are estimated. For this purpose, three methods are compared: the Perturbation Stochastic Finite Element Method (PSFEM) [2], the spectral Stochastic Finite Element Method (SSFEM) [3] and Monte-Carlo method. While these methods have been extensively applied to the resolution of static problems in the literature, few studies have been carried out on their application to eigenproblems. Moreover, due to the nature of the random parameters, i.e. geometric, it is chosen to use an X-FEM approach. X-FEM is naturally associated with a Level Set [4] description of the geometry to provide an efficient treatment of problems involving complex random geometries. The description of the geometry is represented by the zero iso-contour of an implicit function called the Level Set function. This method is very convenient to model random shape variations of a structure. Random variables are shape parameters of basic geometric features which are described with a Level Set representation and/or the control points of the NURBS curves describing the Level Set. One main advantage of this formulation is that the number of random variables remains small. Moreover, X-FEM allows to avoid the remeshing of the structure, which is inevitable when using a classical FEM.

X-FEM is advantageously used in conjunction with the three stochastic methods. First, the perturbation stochastic finite element method is used in order to determine the mean and the variance of natural frequencies. PSFEM consists in a deterministic analysis complemented by a sensitivity analysis with respect to the random parameters. In this framework, X-FEM combined with the Level Set geometry description permits an efficient computation of the structural matrix derivatives. Then, in SSFEM,

since the analytical expression of the structural matrices as a function of the random variables is not always available, a sampling approach is used to determine their projection onto the stochastic basis, i.e. the Polynomial Chaos expansion. In this case, which requires to sample structural matrices for different geometries as in Monte-Carlo method, X-FEM allows sample generation without remeshing the structure.

First, the stochastic methods are investigated on the analysis of the natural frequencies of a beam whose vibrating thickness is considered as a random variable. This simple case for which an analytical solution is available allows the validation of the developed methodology. Then, the modal analysis of a structure whose complex geometry is described by a NURBS curve is carried out. The conclusion drawn from these applications is twofold. First, the advantages and drawbacks of the different methods are highlighted, so that the most efficient method can be selected. Second, these numerical examples illustrate how the stochastic methodology allows the quantification of the influence of uncertain geometric variations on the modal response of resonators.

REFERENCES

[1] N. Moes, J. Dolbow and T. Belytschko. “A finite element method for crack growth without remeshing”. *International Journal for Numerical Methods in Engineering*, Vol. **46**, 131-150, 1999.

[2] M. Kleiber and T.D. Hien. *The stochastic finite element method: basic perturbation technique and computer implementation*, Wiley and Sons, Chichester, 1992.

[3] R.G. Ghanem and P.D. Spanos. *Stochastic Finite Elements: A Spectral Approach*, Springer, Berlin, 1991.

[4] S. Osher and J.A. Sethian. “Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton-Jacobi formulations”. *Journal of Computational Physics*, Vol. **79**, 12-49, 1988.