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TIME-ACCURATE, HIGH-ORDER SCHEMES FOR THE NUMERICAL SIMULATION OF MULTIPHASE FLOW IN POROUS MEDIA

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ABSTRACT

The governing equations of multiphase flow through porous media are often expressed as a coupled nonlinear system: a near-elliptic equation for the pressure or stream function and a quasi-hyperbolic equation for the saturation. After introducing a suitable spatial discretization, the resulting semidiscrete problem can be analyzed as a system of index-2 differential-algebraic equations (DAE). Furthermore, capillary effects and advanced non-equilibrium models induce higher order terms (eventually up to fourth order), which increase the complexity and stiffness of the numerical model.

While a first-order coupling in time, such as the popular IMPES approach, is a widespread and computationally convenient option, the growing interest on high-fidelity simulations, which may shed light on fundamental aspects of porous media flows, requires the development of accurate and efficient high-order time discretizations that honor the fully coupled nature of the problem.

We present high-resolution numerical simulations of advanced continuum models of multiphase flow through porous media. The spatial discretization is performed using a rational spectral method or a weighted essentially non-oscillatory (WENO) scheme, depending on the ability of the computational grid to resolve the complex features of the flow. The discretization in time is carried out by a spectral deferred correction method, which is suitable for stiff differential-algebraic equations. An interesting feature of the deferred correction approach is that our high-order time integration schemes are constructed from low-order approximations, and therefore existing first-order codes can be upgraded to high-order accuracy with reasonable additional coding effort.

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