8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A Bayesian inference approach for structural dynamic transfer function identification

*** Erliang ZHANG[1], Pierre FEISSEL[1], Jérôme ANTONI[1], Claude BLANZ[2]**

[1]Laboratoire Roberval de Mécanique,
Université de Technologie
Compiègne, France
erliang.zhang@utc.fr

[2]Conservatoire National des
Arts et Métiers
Paris, France
claude.blanze@cnam.fr

Key Words: *Transfer function, Bayesian approach, Polynomial Chaos, Evolutionary Markov Chain Monte Carlo*

ABSTRACT

The characterization of dynamic behaviour of the structures (e.g. car engines) plays a vital role in the response prediction, health monitoring, the control of acoustic standard, and so on. The Frequency Response Function (FRF) is one way to describe such behaviour and can be used for identifying a load on structures. The variation of structural parameters of a family of structures leads to the uncertainty of FRF which is described through probabilistic models that can be tuned when dealing with a specific structure, thanks to some measurements. The enhanced characterization of FRF can provide a foundation for the source identification on other structures.

The Bayesian framework is adopted to that purpose since it offers a rigorous foundation for inference from noisy data and uncertain forward models, a natural mechanism for incorporating prior information, and a quantitative assessment of uncertainty in the inferred results [1]. The probability distributions of the parameters of transfer function, where natural frequency ω, damping ratio η and modal residue are of interest, can be updated by the prior ones combining experimental data within the Bayesian framework. The all prior information given is listed as following:

- the measurements of the load F and the response Y on a frequency range $[\omega_i, \omega_s]$
- an additive noise model of the system is adopted based on FRF H:

$$Y(\omega) = H(\omega; \underline{\alpha}_H))F(\omega) + N(\omega) \quad \omega \in [\omega_i, \omega_s] \tag{1}$$

- a prior information on $\underline{\alpha}_H$, described through its probability density.
- a prior information about the uncertainties on the measurement and the lack of accuracy of the model, described through the normal distribution of N.

One of the greatest difficulties is how to construct the prior information about the numerous correlated modal parameters $\underline{\alpha}_H$ of real complicated structures. The uncertainty of modal parameters physically

comes from the variation of structural parameters like material and geometrical parameters. The prior information of $\underline{\alpha}_H$ is therefore constructed based on the uncertain parameters of the structural system through a finite element model, where uncertain model parameters $\underline{\alpha}_m$ are described through a Polynomial Chaos expansion which is an effective way for propagating the uncertainty through the forward model[2]. The PC expansions of $\underline{\alpha}_H$ are hereby obtained through a non-intrusive formulation. Finally, a surrogate posterior density distribution can be derived from the one of model parameters by substituting all the random variables with their expansions of polynomial chaos within the Bayesian framework, which makes sampling from posterior probability distribution be inexpensive.

Sampling from the surrogate posterior probability distribution to obtain the posterior probability distributions of modal parameters is another central task. However the surrogate posterior density distribution is usually of multi-modes because of insufficient data relative to the desired model complexity. An evolutionary Markov chain and Monte Carlo method based on population [3] is adopted, which has the effective proposal mechanism and mixing behaviour. In addition, the optimal values of modal parameters are given as the byproduct of the evolutionary Markov Chain Monte Carlo method and their marginal posterior probability distributions are estimated by kernel density estimation.

modal parameters	**prior values**	**true values**	**posterior values**	**relative errors(per 100)**
ω_1	14.45	16.36	16.36	$\leq 10^{-2}$
ω_2	90.55	102.53	102.53	$\leq 10^{-2}$
ω_3	253.60	287.14	287.15	$\leq 10^{-2}$
ω_4	497.30	563.07	563.09	$\leq 10^{-2}$
ω_5	823.36	932.26	932.28	$\leq 10^{-2}$
ω_6	1233.50	1396.63	1396.65	$\leq 10^{-2}$
η_1	[0, 0.1]	6.4e-4	5.4e-4	15.6
η_2	[0, 0.1]	3.8e-4	4.0e-4	5.3
η_3	[0, 0.1]	8.3e-4	9.3e-4	12.0
η_4	[0, 0.1]	1.6e-3	1.8e-3	12.5
η_5	[0, 0.1]	2.6e-3	2.9e-3	11.5

Table 1: Results of identification of first six modal parameters for cantilever beam, where elastic modulus and density are uncertain, level of noise: SNR = 20dB. unit of ω: Hz

The identification of transfer function is illustrated with an numerical example of cantilever beam (see table 1). The results indicate that Bayesian inference provides a strong framework for the problem of the inference of transfer function, the optimal transfer function and its probability distribution provide the foundation for the source identification within the Bayesian framework.

REFERENCES

[1] A. Tarantola. *Inverse problem theory and methods for model parameter estimation*,SIAM, 2007.

[2] Y.M. Marzouk and H.N. Najm. “Stochastic spectral methods for efficient Bayesian solution of inverse problem”. *Journal of Computational Physics*, Vol. **224**, 560–586, 2007.

[3] F. Liang and W.H. Wong. “Real-parameter evolutionary Monte Carlo with applications to Bayesian mixture models”. *Journal of American Statistical Association*, Vol. **96**, 653–665, 2001.