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Computational Aeroelasticity based on Bifurcation Theory

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ABSTRACT

Developments in computational aeroelasticity have concentrated on time–domain based approaches. Advancing a coupled full–order aerodynamic/structural system in time gives information about the system's stability due to the response to an initial disturbance. To overcome the significant computational costs, in particular to solve for the unsteady, non–linear transonic aerodynamics, alternative approaches involving reduced–order modelling have been considered[1]. The method employed in this work uses the concepts of dynamic systems theory to characterize the stability of complex systems. The BIFOR solver is based on the bifurcation theory and consists of three main parts; i.e. a full–order high–fidelity steady state solver (currently based on the Euler equations), a shifted inverse power method (IPM) algorithm and a Newton eigenvalue (NEV) solver. The IPM provides an appropriate initial guess of the eigenpair as input to the NEV which then calculates the eigenvalues of the Jacobian matrix of the aerodynamic/structural equilibrium solution for the different values of a bifurcation parameter. According to dynamic systems theory, stability is determined by a negative real part for all the Jacobian matrix's eigenvalues. The calculation of the instability point is necessary but knowledge of the type of the instability is also required. Therefore, an extension to the BIFOR solver utilizing the center manifold theory has the ability to predict limit–cycle behavior after the bifurcation[2].

An application considers the 'typical section' airfoil. The Isogai test case[3], which examines a NACA 64A010 airfoil at zero mean angle of attack, is a benchmark case used to demonstrate the performance of a numerical scheme in characterizing the stability behavior. Figure 1 presents a comparison between results of different numerical methods. While there is an overall good agreement of the BIFOR results compared to the other numerical solutions, the superiority of the current method is reflected by the CPU performance (resolution of the instability boundary). Simulations of the stability behavior at 200 Mach numbers were conducted to form the illustrated curve in about 3.5 hours of CPU time on a single processor. However, a numerically created smooth oscillatory trend in the instability boundary has been observed starting for Mach numbers higher than the critical Mach number. This has been discussed for a NACA 0012 airfoil configuration in the current study (Fig. 2). While the Mach number increment can be decreased easily by orders of magnitude (and thus approaches to be continuous), the formed shock wave along the airfoil can not move continuously but is restricted to the grid resolution. Steady state flow characteristics as well as time–marching results have been investigated. The fine resolution of the instability boundary realized in this work means that this oscillatory effect has not been reported previously.

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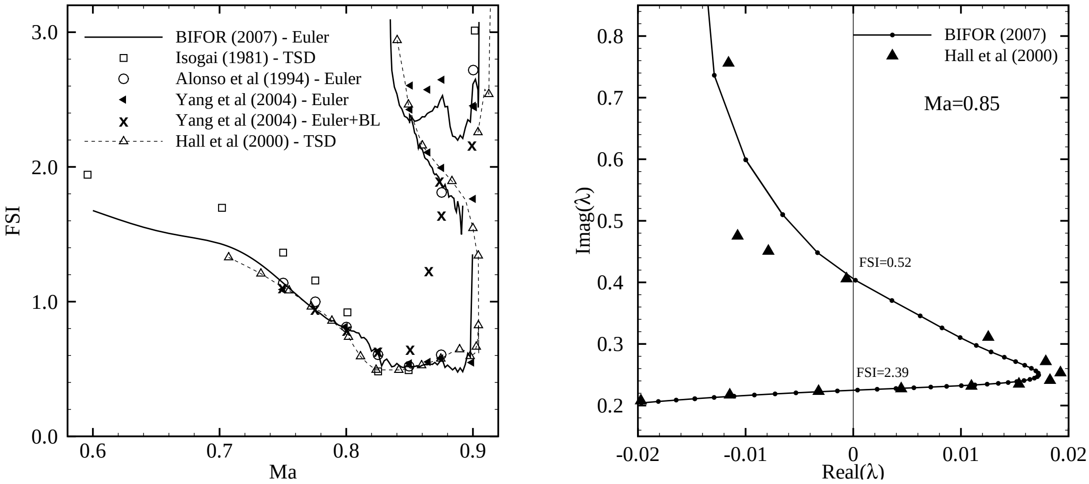
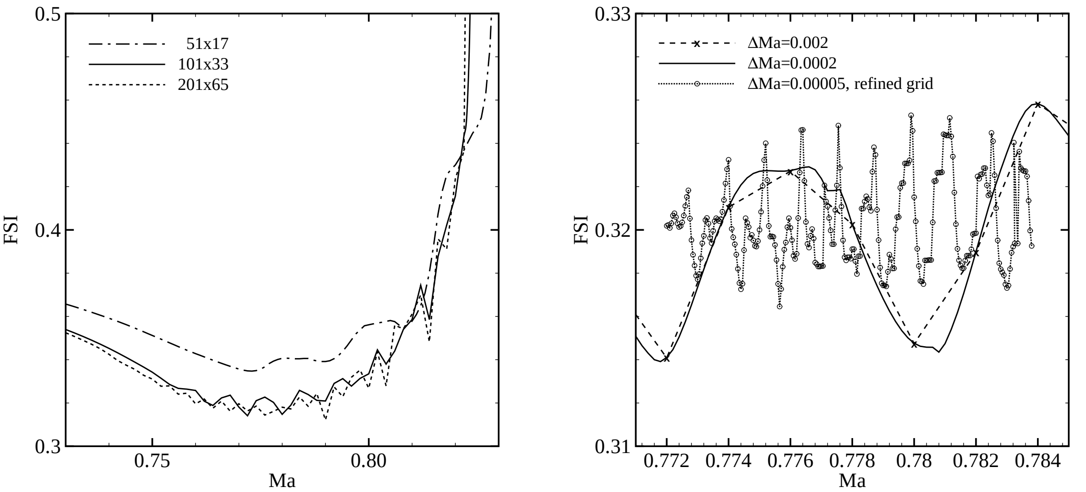


(a) Instability boundary

(b) Root loci compared to Hall et al[4]

Figure 1: Isogai test case



(a) Instability boundary – grid influence

(b) Close–up view

Figure 2: Oscillatory instability boundary