

## Edge-based implementation for the NSGS method

\* Isaac P. Santos<sup>1</sup>, Lucia Catabriga<sup>1</sup> and Regina C. Almeida<sup>2</sup>

<sup>1</sup> Universidade Federal do Espírito Santo  
Av. Fernando Ferrari, 514, 29075-910, Vitória,  
ES, Brazil  
{ipsantos,luciac}@inf.ufes.br

<sup>2</sup> Laboratório Nacional de Computação Científica  
Av. Getulio Vargas, 333, 25651-075, Petrópolis, RJ,  
Brazil  
rcca@lncc.br

**Key Words:** *nonlinear subgrid viscosity, subgrid scale model, data structures for finite element analysis, Krylov iterative methods.*

### ABSTRACT

The Nonlinear Subgrid Stabilization (NSGS) method presented in [3, 4, 5] is a methodology for approximating transport equations, based on a two level decomposition of the approximation space. The two-scale viscosity model is obtained by adding to the Galerkin formulation a nonlinear operator acting only on the unresolved mesh scales. The key feature is the local control coming from the decomposition of the velocity field into the resolved and unresolved scales and requiring the satisfaction of the discrete model problem at the element level for a minimum kinetic energy associated to the unresolved scales. The amount of subgrid viscosity is adjusted according to the residual of the solution on the coarsest refined mesh. That is, it does not depend on the choice of any stabilizing parameter.

In the present work, the computational aspects of the NSGS method are addressed. We perform a study of required data structures and the algebraic system is solved by Krylov space iterative update techniques. Both element-based and edge-based data structures are considered. The success of this solution strategy requires an efficient implementation of matrix-vector products and the choice of a suitable preconditioner.

Element-based data structures have been extensively used in the finite element implementation and a wide class of preconditioners specially designed for element-by-element (EBE) techniques can be found in the literature. The matrix-vector products and right-hand side evaluations can be written as a single loop structure when this kind of data structure is used. Edge-based data structures (EDE) has been used since the early 1990s. The resulting element matrices and residual terms are disassembled into their edge contributions [1, 2]. Thus, matrix-vector products and right-hand side evaluations may be computed sweeping through edges rather than elements. The EDE format can be viewed as a blend of EBE and CSR formats: only global nonzero coefficients are stored and a single loop structure is used to compute the matrix-vector products. Generally, the edge-based data structure reduces processing time and requires around one half of the storage area to hold the coefficient matrix when compared to an enhanced EBE implementation.

Our studies include comparisons between the element-by-element and edge-based data structures for benchmark 2D problems considering a variety of numerical experiments which cover the regimes of dominant advection and dominant absorption.

## References

- [1] L. Catabriga and A.L.G.A. Coutinho. Implicit SUPG solution of Euler equations using edge-based data structures. *Computer Methods in Applied Mechanics and Engineering*, 191:3477–3490, 2002.
- [2] L. Catabriga, A.L.G.A. Coutinho, and T.E. Tezduyar. Compressible flow SUPG stabilization parameters computed from element-edge matrices. *Computational Fluid Dynamics Journal*, 13:450–459, 2004.
- [3] I. P. Santos and R. C. Almeida. A free parameter subgrid scale model for transport problems. *Computer Methods in Applied Mechanics and Engineering* - (submitted).
- [4] I. P. Santos and R. C. Almeida. Numerical analysis of the nonlinear subgrid scale model. (submitted).
- [5] I. P. Santos and R. C. Almeida. A nonlinear subgrid method for advection-diffusion problems. *Computer Methods in Applied Mechanics and Engineering*, 196:4771–4778, 2007.