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NON-DIRECT DETERMINATION OF ADHESIVE PROPERTIES OF ELASTIC MATERIALS BY DEPTH-SENSING INDENTATION

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ABSTRACT

Depth-sensing indentation of samples by sharp indenters is currently a routine technique for extracting elastic characteristics of materials. The load-displacement curve at loading usually reflects both elastic and plastic deformations of the material because high pressures under sharp indenters cause normally plastic deformations of the material. Therefore, elastic characteristics are usually estimated using unloading branch of the curve and accepting the following assumptions: (i) the material deforms elastically at unloading, (ii) the formulae derived from the Hertz contact theory are valid, and (iii) influence of adhesion can be neglected [3]. So far the inverse analysis of depth-sensing indentation experiments dealt with sharp indenters [1]. However, the drawbacks of the employment of the sharp indenters are well known [3, 4]: strictly speaking, the assumptions of the Hertz contact theory are not valid for contact between a sharp indenter and a plastically deformed surface of the material sample having an inhomogeneous field of residual stresses. Hence, it has been suggested to determine elastic constants of materials from the Hertz formulae for spherical indenters using the initial elastic stage of the load-displacement curves [4]. However, if a material is tested by a spherical indenter then at low applied loads the measurements may be greatly influenced by adhesion of materials. Values of the adhesive force are currently determined only from direct measurements [6]. However, direct measurements of the adhesive force and the minimum value of the displacement by the load-displacement diagram are rather difficult. The difficulty is caused not only by the restricted precision of the measurement devices, but also by instability of the diagrams for a real experimental device at ultra-low tensile loads. Techniques that have been developed for determination of elastic characteristics from direct measurements of the pull-off force [6], are valid only for experimental conditions when the JKR theory is applicable. In addition, the tensile (adhesive) part of the load-displacement diagram may be greatly influenced by roughness. It is known for non-adhesive contact problems that the trend of the compressive part of the load-displacement diagram is independent of fine distinctions between functions describing roughness [2]. One can assume that the compressive part of the load-displacement diagram for adhesive contact is also much less sensitive to roughness than the tensile part of the diagram.

In this presentation it is shown the principal possibility of extracting the adhesive characteristics from the depth-sensing indentation diagrams employing a non-direct approach, i.e. by solving an inverse problem without direct measurements of the adhesive force. It is considered the problem of adhesive contact between an elastic sphere and a flat surface of an elastic sample, and corresponding load-displacement curves $P{-}\delta$. The new non-direct approach is based on mathematical techniques of solving ill-posed problems and the approximation of the experimental data on the stable stages of the indentation diagrams by appropriate theory of adhesive contact of spheres. The theories of adhesive contact of spheres are represented as a functional relation $P - \delta$ of the following type [5]

$$F\left(\frac{P}{P_c}, \frac{\delta}{\delta_c}, \lambda\right) = 0 \tag{1}$$

where F is given by one of the well-established theories that include JKR, DMT, and Maugis theories, P_c and δ_c are characteristics of scales for P and δ at low loads and small displacements. In the JKR theory P_c is the absolute value of pull-off force and δ_c is the absolute value of the minimum displacement that occurs due to adhesion. For each value of the Tabor-Maugis parameter λ ($\lambda \approx 0.73\delta_c/z_0$ where z_0 is the equilibrium separation between surfaces), the graph of the functional relation $P - \delta$ is situated between the corresponding graphs for the JKR and DMT theories [5]. If (P_i, δ_i), $i = 1, \ldots, N$ are respectively experimental values of the compressing load $P \geq 0$ and corresponding values of the displacement $\delta \geq 0$ then the problem is reduced to determination of $P_c \geq 0$ and $\delta_c \geq 0$ (two unknown values) from the following system of a large number of non-linear equations

$$F\left(\frac{P_i}{P_c}, \frac{\delta_i}{\delta_c}, \lambda\right) = 0, \qquad i = 1, \ldots, N. \tag{2}$$

The system (2) is overdetermined for $N > 2$ and hence, normally there exists no solution in the classic sense. In addition, the experimental data have always some measurement errors. Therefore the determination of the characteristics P_c and δ_c is an ill-posed problem and various regularization techniques are employed. If it is known a priori that the values (P_i, δ_i) are practically exact then one may use the least square method. After the values of the scale characteristics have been extracted from the reliable data, other mechanical and adhesive characteristics are calculated. In particular, the values of the pull-off force, the minimum value of the displacement, the work of adhesion and the reduced elastic modulus of the materials. The main ideas of application of new techniques are demonstrated on examples.

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