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A LEVEL-SET METHOD FOR EFFICIENT MONOLITHIC MODELING OF ELECTROMECHANICAL COUPLING

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ABSTRACT

The existing techniques for modelling electrostatic-structural coupling require the finite element mesh of the electric field and the structure to conform, which can cause mesh distortions (Fig. 1).

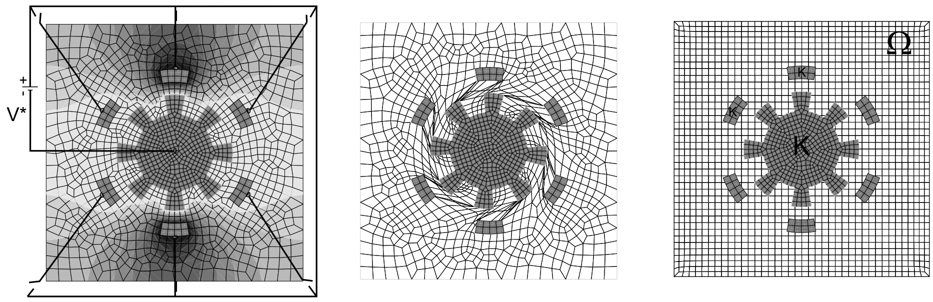

Figure 1: Simulation of an electric motor. Left - initial electric field. Center - distortion of the conforming electric mesh. Right - proposed approach with independent descritizations.

In fluid-structure interaction this problem was circumvented by using a fixed fluid mesh in which the structure mesh is moving. Compatibility between the two domains is provided by a distributed Lagrange multiplier (Glowinski et al. 1994). Legay et al. (2006) enhanced this method by a level set technique, which allows to negate the mass density and the stress of the fluid, overlapping with the solid. This led to a consistent physical description of the problem.

In this work we propose a monolithic finite element formulation for electrostatic-structural coupling, based on the Gibbs energy of the system

$$L = L(\mathbf{u}, \phi, \lambda) = W_k - W_\Omega + W_\lambda, \tag{1}$$

where W_k is the mechanic energy of the conductor, $W_\Omega = \frac{1}{2}\int_\Omega (\nabla\phi)^T \boldsymbol{\varepsilon}_0 \nabla\phi H(f(\mathbf{x}))\mathrm{dV}$ - energy of the electric field and $W_\lambda = \int_{\partial K} \lambda(\phi - \phi^*)\mathrm{dS}$ is the constraint term with Lagrange multiplier that enforces potential ϕ^* on the conductor's boundary. $f(\mathbf{x})$ – is the level set function, defined on Ω; it is positive outside the conductor and negative otherwise. H – is the Heaviside function, equal to unity for positive arguments and zero otherwise (hence, inside the conductor). We have verified that the stationary point of the above functional is the true solution of the corresponding electrostatic-structural problem and thus that the deduced electrostatic force is correctly computed.

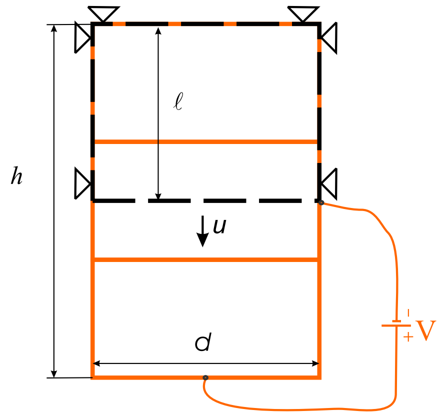

Figure 2: 2D mesh for the one dimensional problem. The structure is represented by one quad element, the electric field – by three. The meshes are non-conforming.

The above level-set approach was implemented in a 2D FE formulation and used to solve a 1D problem of a truss (modelled by one quad element), embedded into a 1D electric field (modelled by 3 quad elements). The structural and electric meshes are not conforming (Fig. 2). It is well-known that voltage controlled simulation of such system has a so-called pull-in point. The proposed finite element formulation allows to apply charge driven boundary conditions, which can be used to simulate the system after the pull-in point was passed (Fig. 3, left). Given the fact that this is an energy based formulation, we could obtain a consistent stiffness of the system that we used to calculate the lowest eigen frequency of the structure as a function of the applied voltage (Fig.3, right). Both, small and finite strain assumptions were used. The results of the small strain simulations were identical to the corresponding analytical solutions. This modelling strategy is very promising

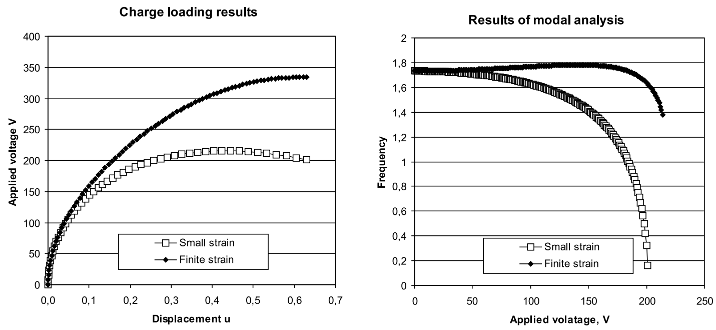

Figure 3: Results of the charge loading (left) and the modal analysis (right).

since it allows keeping the underlying electrostatic mesh unchanged while the structure moves through it. Currently simulations of 2D problems are performed and evaluated.

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