8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

DYNAMICS ANALYSIS OF NON LINEAR UNCERTAIN SYSTEMS BY A JOINED GALERKIN/RBF APPROACH

***M. Betti[1], P. Biagini[2] and L. Facchini[3]**

[1] Univ. of Florence
Via di S. Marta 3
I-50139 Florence (ITALY)
mbetti@dicea.unifi.it

[2] Univ. of Florence
Via di S. Marta 3
I-50139 Florence (ITALY)
paolo.biagini@dicea.unifi.it

[3] Univ. of Florence
Via di S. Marta 3
I-50139 Florence (ITALY)
luca.facchini@unifi.it

Key Words: *Parameter uncertainties, Non-linear dynamics, Galerkin methods.*

ABSTRACT

The analysis of uncertain systems is becoming an actual engineering concern. Frequently it is necessary to compute response quantities (displacement, stress state, frequencies, etc.) in mechanical or structural systems whose characteristics depend on a set of not deterministically known parameters (f.i. manufacture errors or inaccuracy in measurement of system properties). These variations could lead to large and unexpected excursions of the structural response and may lead to drastic reductions in structural safety. A probabilistic approach is necessary for adequate reliability analysis. The problem may be faced by means of Montecarlo simulation, which allows statistical evaluation after a large number of analyses with different values of the random parameters. This approach is very computationally expensive, especially when the phenomena to be investigated are controlled by non-linear equations. For this reason some non-statistical alternative procedures have been proposed. These alternative procedures mainly consist in a direct approach using probabilistic, instead of statistic, theory. This is usually pursued in both static and dynamic setting by using expansion methods, where the stiffness matrix of the structural problem is split in a deterministic part (obtained with the mean value of random parameters) and a part which accounts for the fluctuation of the random variables about its mean value.

In order to evaluate the probabilistic response, Taylor or Neumann or Galerkin expansions ([1], [2], [3]) are adopted to avoid inversion of matrices depending on the random parameters. These methods in conjunction with discretization of random fields using Karhunen-Loeve expansion ([4]) or spectral approach ([3], [5]) allow obtaining approximate solutions in terms of moments of the response. In this line of research, the paper aim to offer a method to solve dynamic problems involving parameter uncertainties. The proposed procedure can be viewed as a conditional analysis where the dependence of the structural response on the uncertain parameters is modelled by means of a Galerkin method whose trial functions are chosen among radial basis functions ([6], [7], [8]). In particular, assuming an equation of motion of a non-linear system in the following form:

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{g}(\mathbf{b}, \mathbf{x}, \dot{\mathbf{x}}) = \mathbf{F}(t) \qquad (1)$$

where the mass **M**, the non-linear function **g**, and the external forcing process **F** may depend on random parameters which can be grouped in the vector **b**. The solution will be function of time *t* and of the vector of uncertain parameters **b**. Such dependence is modelled as follows:

$$x(t,\mathbf{b}) = \sum_{k=1}^{N_\varphi} w_k(t)\varphi_k(\mathbf{b}) \tag{2}$$

where $w_k(t)$ are time-dependent coefficients to be determined, while $\varphi_k(\mathbf{b})$ are the trial functions chosen among radial basis functions of the random parameters. Each generic radial basis function is characterised by its centre $c^{(k)}$ and its decay parameter σ:

$$\varphi_k(\mathbf{b}) = \phi\left(\left\|\mathbf{b} - \mathbf{c}^{(k)}\right\|\right) \tag{3}$$

Main advantages of the considered approach are listed in the following: a) the number of trial functions can be kept as low as possible by optimization of the radial basis functions to reduce the computation effort; b) an error parameter can be defined, so that the quality of the approximating expression (2) can be checked at all instants. The paper present results of the proposed approach developing some illustrative case studies; on the case studies is discussed with emphasis the criteria, based on the error parameter, to accept or reject the obtained solution. Results are then compared by Montecarlo simulation.

REFERENCES

[1] W.K. Liu, T. Belytschko and A. Mani, "Probabilistic finite elements for nonlinear structural dynamics", *Computer Methods in Applied Mechanics and Engineering*, Vol. **56**, pp. 61-81, (1996).

[2] R.G. Ghanem and P.D. Spanos, "A stochastic Galerkin expansion for nonlinear random vibration analysis", *Probabilistic Engineering Mechanics*, Vol. **8**, pp. 255-264, (1993).

[3] P.D. Spanos and R.G. Ghanem, "Stochastic finite element expansion for random media", *Journal of Engineering Mechanics (ASCE)*, Vol. **115**, pp. 1035–1053, (1989).

[4] R.G. Ghanem and P.D. Spanos, *Stochastic Finite Elements: a Spectral Approach*, New York: Springer-Verlag, 1991.

[5] R.G. Ghanem and P.D. Spanos, "Spectral stochastic finite-element formulation for reliability analysis", *Journal of Engineering Mechanics (ASCE)*, Vol. **117**, pp. 2351–2372, (1991).

[6] T. Poggio and F. Girosi, "Networks for approximation and learning", *Proc. IEEE, 78*, pp. 1481-1497, (1990).

[7] D.S. Broomhead and D. Lowe, "Multi-variable functional interpolation and adaptive networks", *Complex Syst.*, Vol. **2**, pp. 269-303, (1988).

[8] B. Gotovac and V. Kozulic, "On a selection of basis functions in numerical analyses of engineering problems", *International Journal for Engineering Modelling*, Vol. 12, pp. 25-41, (1999).