

ROBUST STOCHASTIC SYSTEM DESIGN: OPTIMIZATION TREATING MODEL UNCERTAINTY

*James L. Beck¹ and Alexandros A. Taflanidis²

¹ California Institute of Technology
Engineering and Applied Science Division
1200 E California Blvd MC 104-44
Pasadena, CA 91125
jimbeck@caltech.edu

² Duke University
Civil and Environmental Engineering
Department
Box 90287 Hudson Hall
Durham, NC 27708
alexandros.taflanidis@duke.edu

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ABSTRACT

In engineering design, the knowledge about a planned system is never complete. First, it is not known in advance which design will lead to the best system performance in terms of a specified metric; it is therefore desirable to optimize the performance measure over the space of design variables that define the set of acceptable designs. Second, modeling uncertainty arises because no mathematical model can capture perfectly the behavior of a real system and its environment. In practice, the designer chooses a model that he or she feels will adequately represent the behavior of the built system as well as its future excitation; however, there is always uncertainty about which values of the model parameters will give the best representation of the constructed system and its environment, so this parameter uncertainty should be quantified. Furthermore, whatever model is chosen, there will always be an uncertain prediction error between the model and system responses. For an efficient engineering design, all these uncertainties, associated with future excitation events, as well as the modeling of the system, must be explicitly accounted for.

A probability logic approach [1] provides a rational and consistent framework for quantifying all aforementioned uncertainties. In this approach, probability can be interpreted as a means of describing the incomplete, i.e., missing, information about the system under consideration. This is established by characterizing the relative plausibility of different properties of the system and future excitations by probability models. These probability models incorporate into the modeling process our available knowledge about the system and its environment. The design objective in this stochastic framework is related to the expected value of a system performance measure, such as reliability or expected life-cycle cost, representing the expected utility from a decision-theoretic point of view. In this case, the system design process is often called *stochastic system design* and the associated design optimization problem *stochastic design optimization*.

Such design problems are challenging, because of the complex coupling between (i) system modelling (including quantification of uncertainties), (ii) performance quantification, (iii) stochastic analysis (propagation of uncertainties), and (iv) design optimization. This study discusses initially the characteristics of all these distinct tasks and the connection that exists among them. It then focuses in detail on two important aspects. The first of these (stochastic performance evaluation) involves the computation of the expected value corresponding to the design performance objective, and the second (design optimization) involves the optimization of that expected value over the space of the admissible values for the design variables of the problem. For complex system models, this expected value can rarely be evaluated, or efficiently approximated, analytically. To address this issue, calculation by means of stochastic simulation techniques is used. This approach, though, involves an unavoidable estimation error and significant computational cost, features which make the associated stochastic design optimization challenging. An efficient framework consisting of two stages is described for this optimization. The first stage implements a novel approach, called Stochastic Subset Optimization (SSO) [2], for iteratively identifying a subset of the original design space that has high plausibility of containing the optimal design variables. The second stage adopts some other stochastic optimization algorithm to pinpoint the optimal design variables within that subset.

An illustrative example is presented that shows the efficiency of the proposed methodology; it considers the seismic retrofitting of a four-story non-ductile reinforced-concrete building with linear viscous dampers. Minimization of the expected life-cycle cost that includes structural modelling uncertainties and seismic excitation uncertainties is adopted as the design objective. A comprehensive loss estimation methodology is utilized for estimating the repair costs due to damage from future seismic events; this methodology [3] uses the nonlinear time-history response of the structure to seismic excitation to estimate the damage at a detailed structural and non-structural component level. Realistic models are used for the structural behavior as well as for describing future ground motions that probabilistically take into account all important uncertainties. The maximum force capacities of the dampers placed on each floor correspond to the design variables for the problem. For this example, the expected life-cycle cost with the optimal dampers installed is much less than that for the original un-retrofitted building.

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